# SOLUZIONI ESAME 12-02-2018

## ES1

Laser He-Ne

$m_{He} = 0.66 \times 10^{-26}$ kg

$m_{Ne} = 3.35 \times 10^{-26}$ kg

$\lambda_0 = 543.5$ nm

$\gamma_S = 0.04$ $m^{-1}$

$\gamma_T = 0.092$ $m^{-1}$

$d = 1$ mm

$\Delta\nu_{FSR} = 500$ MHz

$P_{OUT} = 2$ mW

$N_{PH} = 1.55 \times 10^{15}$ $m^{-3}$

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a) Nota $T = 150\,°C$, calcolare $\Delta\nu_{FWHM}$ per effetto Doppler, e il numero di modi oscillanti

$$\Delta\nu_{FWHM} = 2\nu_0\sqrt{\frac{2kT\ln(2)}{m_{Ne}c^2}} \longrightarrow \begin{cases} \nu_0 = \dfrac{c}{\lambda_0} = 551.6 \text{ THz} \\ T = 150 + 273.15 = 423.15 \text{ K} \end{cases}$$

$$\rightarrow \Delta\nu_{FWHM} = 1.81 \text{ GHz}$$

$$M = \frac{\Delta\nu_{FWHM}}{\Delta\nu_{FSR}} = 3.62 \longrightarrow 3 \text{ o } 4 \text{ modi oscillanti}$$

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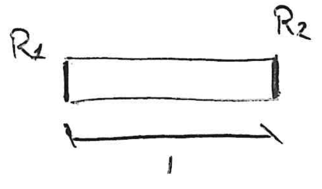
b) Determinare $L$

$\Delta\nu_{FSR}$ è noto



$$\Delta\phi_{RT} = 2m\pi$$

$$\rightarrow \frac{2\pi}{\lambda}\cdot 2L = 2m\pi \rightarrow \nu_m = m\frac{c}{2L}$$

$$\Delta\nu_{FSR} = \frac{c}{2L} \quad \rightarrow \quad L = \frac{c}{2\Delta\nu_{FSR}} = 29.98\ cm$$

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c) Calcolare $R_1$ e $R_2$

È nota $P_{out}$: $P_{out}$ è proporzionale alla trasmittanza dello specchio d'uscita

Assumendo $R_1 = R_{out}$, $\rightarrow P_{out} = \phi \rightarrow h\nu_0 A (1-R_1)$

$$\rightarrow P_{out} = \frac{1}{2} N_{PM}\, c\, h\nu_0 A (1-R_1)$$

$$(1-R_1) = \frac{P_{out}}{\frac{1}{2} N_{PM}\, c\, h\nu_0 A} = 0.03$$

$$A = \frac{\pi d^2}{4}$$

$$\rightarrow R_1 = 97\%$$

Noto $R_1$ e note le perdite interne e totali, trovo $R_2$

$$a_T = a_S + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right)$$

$$R_2 = \frac{1}{R_1} e^{-2L(a_T - a_S)}$$

$$R_2 = 99.93\%$$

## ES 2 — Sistema di comunicazione in fibra ottica

$\lambda_0 = 1310$ nm

FIBRA OTTICA

$n_1 = 1.45$

$r = 3\ \mu m$

$\alpha_F = 0.8 \frac{dB}{km}$

$L_s = 1 km$

$\alpha_J = 1.1 dB$

LED
Lambertiano

$P_{LED} = 1 mW$

DETECTOR

$S = 100 nW$

$\alpha_{DET} = 3 dB$

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a) $n_2 = 1.4$, calcolare $\lambda_{cut-off}$

$$V = \frac{2\pi r}{\lambda} NA$$

$$\lambda_{cut-off} = \frac{2\pi r\, NA}{2.405} \quad , \quad NA = \sqrt{n_1^2 - n_2^2} = 0.3775$$

$$\lambda_{cut-off} = 2.96\ \mu m$$

→ Propagazione multimodale in tutte le tre finestre di trasmissione (850 nm, 1310 nm, 1550 nm)

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b) Riprogettare $n_2$

1) $V|_{1310nm} \leq 2.405$

$$\frac{2\pi r}{\lambda} NA \leq 2.405 \rightarrow NA \leq \frac{2.405 \cdot 1310 nm}{2\pi \cdot 3 \mu m} = 0.1671$$

2) $\alpha_{LF} \leq 20\,dB$

LED è lambertiano: $\frac{P_{FIBRA}}{P_{LED}} = NA^2$

$$\alpha_{LF} = 10\log_{10}\left(\frac{P_{LED}}{P_{FIBRA}}\right) = 10\log_{10}\left(\frac{1}{NA^2}\right)$$

→ $\alpha$ è definita come <u>un'attenuazione</u>

$$10\log_{10}\left(\frac{1}{NA^2}\right) \leq 20$$

$$NA^2 \geq 10^{-2}$$

$$NA \geq 0.1$$

$$\Rightarrow \quad 0.1 \leq NA \leq 0.1671$$

Scelgo ad esempio $NA = 0.15$ → $n_2 = 1.442$

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c) Determinare $L_{MAX}$


N spezzoni

(N-1) giunzioni

power budget:

$$10\log_{10}\left(\frac{P_{LED}}{1mW}\right) - \alpha_{LF} - \alpha_F \cdot L_s \cdot N - \alpha_J(N-1) - \alpha_D \geq 10\log_{10}\left(\frac{S_{DET}}{1mW}\right)$$

$$\alpha_{LF} = 10\log_{10}\left(\frac{1}{NA^2}\right) = 16.48\,dB$$

$$0\,dBm - (-40\,dBm) - 16.48\,dB - 0.8\,dB \cdot N - 1.1\,dB(N-1) - 3\,dB \geq 0$$

$$40\,dB - 16.48\,dB + 1.1\,dB - 3\,dB \geq N(1.9\,dB)$$

$$N \leq 11.38$$

$$\Rightarrow N_{MAX} = 11 \quad \rightarrow \quad L_{MAX} = 11\,km$$

ES 3

Si RAPD

$W_\pi = 20\mu m$

$N_{A,\pi} = 5\times10^{13}\ cm^{-3}$

$W_{p,av} = 700\ nm$

$N_{A,av} = 2\times10^{16}\ cm^{-3}$

$W_{n^+} = 300\ nm$

AR in $TiO_2$, $n_{TiO_2} = 2.5$

$\lambda_0 = 800\ nm$

$\alpha = 10^3\ cm^{-1}$

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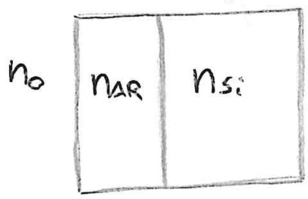
a) Calcolare T con $TiO_2$ AR coating




$$n_{Si} = \sqrt{\varepsilon_r} = 3.42$$

$$R = \left(\frac{n_0 n_{Si} - n_{AR}^2}{n_0 n_{Si} + n_{AR}^2}\right)^2 = 8.56\%$$

$$T = (1-R) = 91.4\%$$

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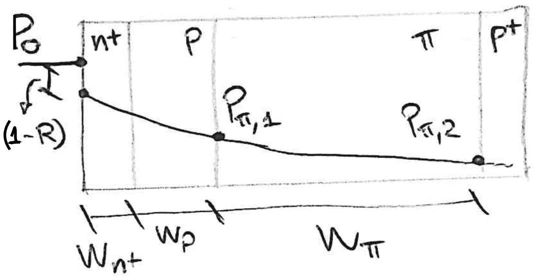
b) $\eta_\pi$ : frazione di potenza assorbita in $\pi$




$$\eta_\pi = \frac{P_{\pi,1} - P_{\pi,2}}{P_0}$$

$$\eta_\pi = \frac{P_0(1-R)\left[e^{-\alpha(W_{n^+}+W_p)} - e^{-\alpha(W_{n^+}+W_p+W_\pi)}\right]}{P_0}$$

$$\eta_\pi = (1-R)\, e^{-\alpha(W_n+W_p)}\left(1 - e^{-\alpha W_\pi}\right)$$

$$\eta_\pi = 0.715$$

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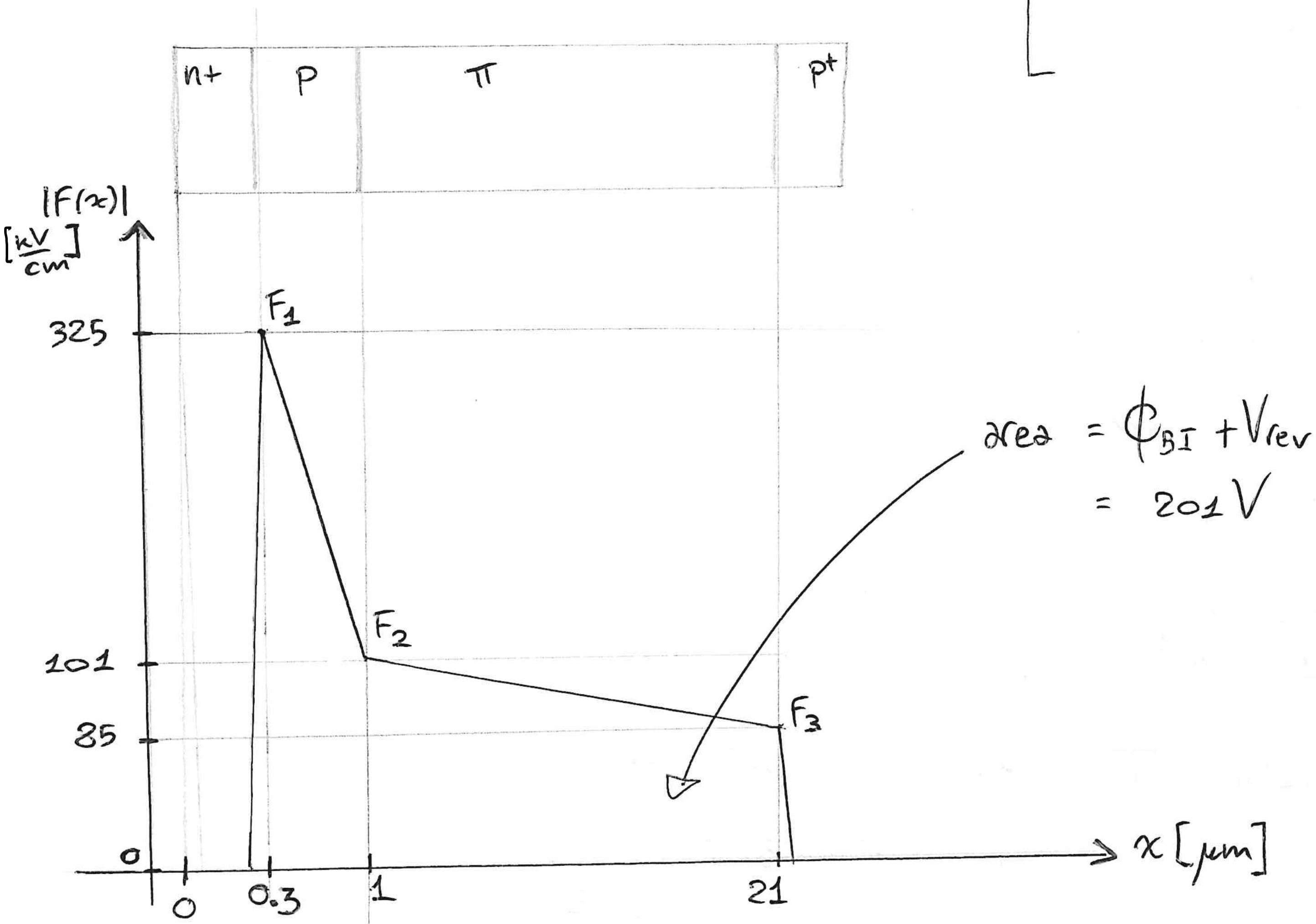
c) $V_{rev} = 200V$, grafico quotato di $F(x)$

[Trascurare il depletion layer in $n^+$ e $p^+$]




$$\begin{cases} \dfrac{F_1 + F_2}{2} W_p + \dfrac{F_2 + F_3}{2} W_\pi = 201\,V \\ F_2 = F_1 - q\dfrac{N_p}{\varepsilon} W_p \\ F_3 = F_1 - q\dfrac{N_p}{\varepsilon} W_p - q\dfrac{N_\pi W_\pi}{\varepsilon} \end{cases}$$

$(\varepsilon = \varepsilon_0 \varepsilon_r)$

$$\frac{F_1}{2} W_p + \frac{F_1}{2} W_p - q\frac{N_p}{2\varepsilon} W_p^2 + \frac{F_1}{2} W_\pi - q\frac{N_p W_p}{2\varepsilon} W_\pi + \frac{F_1}{2} W_\pi - q\frac{N_p W_p}{2\varepsilon} W_\pi - q\frac{N_\pi W_\pi^2}{2\varepsilon} = 201\,V$$

$\rightarrow F_1 = 325 \dfrac{kV}{cm}$

$F_2 = 101 \dfrac{kV}{cm}$

$F_3 = 85 \dfrac{kV}{cm}$