SOLUZIONI ESAME 05/09/2018

ES 1

Laser Nd:YAG $(n_{YAG} = 1.82)$

$\lambda_0 = 1064$ nm

Livello laser superiore $E_2 = 1.4271$ eV

$\Delta\nu_{FWHM} = 122$ GHz

Lunghezza barretta cristallina $l = 2$ cm

Lunghezza risonatore $L = 8$ cm

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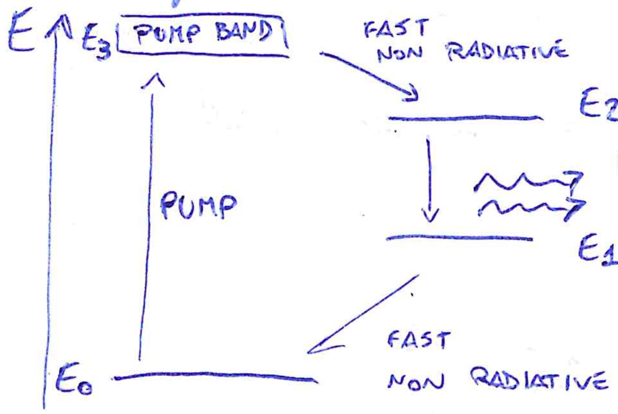
a) Disegnare un diagramma dei livelli energetici e calcolare E_1



$$E_{PH} = E_2 - E_1 = h\nu = \frac{hc}{\lambda_0}$$

$$E_{PH} = 1.1654 \text{ eV}$$

$$E_1 = E_2 - \frac{hc}{\lambda_0} = 0.2617 \text{ eV}$$

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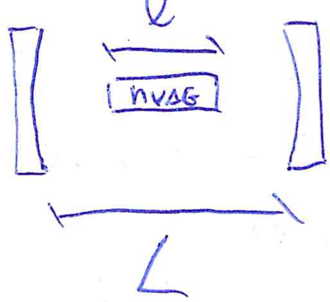
b) Calcolare il numero di modi oscillanti



Si pone lo sfasamento di round-trip pari a $2m\pi$:

$$\Delta\phi_{RT} = k_0 \cdot L_{OTT,RT} = 2m\pi$$

costante di propagazione nel vuoto

cammino ottico di round-trip

$$\frac{2\pi}{\lambda_{0,m}} L_{OTT,RT} = 2m\pi$$

$$\rightarrow \nu_m = m \frac{c}{2 L_{OTT,RT}} \quad ; \quad L_{OTT,RT} = 2\left[n_0 (L-\ell) + n_{YAG}\, \ell \right]$$

$$\rightarrow \Delta\nu_{FSR} = \frac{c}{2\left[L + (n_{YAG}-1)\ell\right]} = 1.556\ GHz$$

$$\#\ \text{modi} = \frac{\Delta\nu_{FWHM}}{\Delta\nu_{FSR}} = 78.4 \rightarrow 78 \ \text{modi}$$

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c) Dimensionare indice di riflessione e spessore dello strato antiriflesso

n_0 | n_{AR} | n_{YAG}

$$R = \left(\frac{n_0 n_{YAG} - n_{AR}^2}{n_0 n_{YAG} + n_{AR}^2} \right)^2$$

$R = 0$ se $n_{AR} = \sqrt{n_0 n_{YAG}} = 1.349$

$$d_{AR} = \frac{m \lambda_0}{4 n_{AR}} = \underset{\uparrow\, m=1}{197\ nm}$$

$\boxed{ES\,2}$

InGaN LED

$T = 300\,K \qquad E_G(300K) = 2.85\,eV$

$\eta_{EQE} = 18\%$

$V_F = 3V$

$$E_G(T) = E_{G,0} - \frac{AT^2}{B+T} \; ; \quad A = 4.62 \times 10^{-4}\,\frac{eV}{K} \quad B = 420\,K$$

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a) Determinare λ_0 e $\Delta\lambda_{FWHM}$

$$E_{PEAK} = E_G + \frac{kT}{2} = 2.863\,eV$$

$$\lambda_0 = \frac{hc}{E_{PEAK}} = \frac{1.24\,eV\text{-}\mu m}{2.863\,eV} = 433\,nm$$

$$\Delta E \cong 3kT = 77.4\,meV \rightarrow \Delta\lambda_{FWHM} = \frac{hc}{E_{PEAK}^2}\,\Delta E_{FWHM} = 11.7\,n$$

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b) Calcolare η_{PCE}, e P_{out} per $I_F = 10\,mA$

$\eta_{EQE} = 18\%$

$$\eta_{EQE} \triangleq \frac{\phi_{PH,out}}{\phi_{el,in}} = \frac{P_{OTT}/h\nu}{I_F/q} = \frac{P_{OTT}}{I_F}\,\frac{q}{h\nu}$$

$$\eta_{PCE} \triangleq \frac{P_{OTT}}{V_F I_F} = \frac{1}{V_F}\,\eta_{EQE}\,\frac{h\nu}{q} \simeq \eta_{EQE}\,\frac{E_{GAP}}{qV_F} = 17.1\%$$

Se $I_F = 10\,mA \rightarrow P_{OTT} = \eta_{PCE} I_F V_F = 5.1\,mW$

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c) Calcolare ΔT che causa $\Delta\lambda_{PEAK} = 2\,nm$

$$\lambda_{PEAK} = \frac{hc}{E_{PEAK}(T)} \rightarrow \frac{d\lambda_P}{dT} = -\frac{hc}{E_P^2}\frac{dE_P}{dT}$$

$$E_P(T) = E_G(T) + \frac{kT}{2} = E_{G0} - \frac{AT^2}{B+T} + \frac{kT}{2}$$

$$\frac{dE_P}{dT} = \frac{-2AT(B+T) + AT^2}{(B+T)^2} + \frac{k}{2}$$

$$= -3.05\times10^{-4}\,\frac{eV}{K} + 0.43\times10^{-4}\,\frac{eV}{K} = -2.62\times10^{-4}\,\frac{eV}{K}$$

- $$\frac{d\lambda_P}{dT} = -\frac{hc}{E_P^2}\frac{dE_P}{dT} = +3.96\times10^{-2}\,\frac{nm}{K}$$

Linearizzando, $\Delta\lambda_{PEAK} \cong \left|\frac{d\lambda_P}{dT}\right| \Delta T$

$$\rightarrow \Delta T = \frac{2\,nm}{\left|\frac{d\lambda_P}{dT}\right|} = 50.5\,^\circ C$$

$\boxed{ES3}$

Cella solare

$I_0 = 1.1\,nA$

$I_{PH} = 18\ mA$ per radiazione incidente $J = 1\frac{kW}{m^2}$

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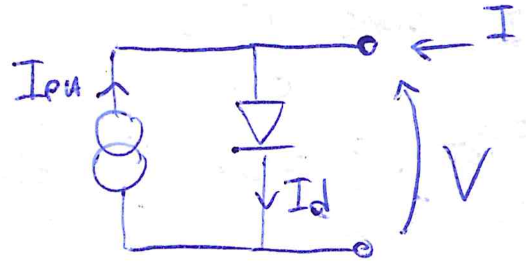
a) Circuito equivalente, I_{SC}, V_{OC}



$$I = -I_{PH} + I_0 \left(e^{\frac{qV}{kT}} - 1\right)$$

$$I_{SC} \triangleq I\Big|_{V=0} = -I_{PH}$$

$$V_{OC} \triangleq V\Big|_{I=0} \rightarrow I_{PH} = I_0 \left(e^{\frac{qV_{OC}}{kT}} - 1\right)$$

trascurabile

$$\rightarrow V_{OC} = \frac{kT}{q} \ln\left(\frac{I_{PH}}{I_0} \cancel{+1}\right) \cong 0.43V$$

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b) I_{SC}, V_{OC} nel caso $J_2 = 250 \frac{W}{m^2}$

La fotocorrente è proporzionale all'intensità della radiazione J

$I_{PH} = \mathcal{C} J$

$$\rightarrow \frac{I_{PH_2}}{I_{PH_1}} = \frac{J_2}{J_1} \rightarrow I_{PH_2} = \frac{I_{PH_1}}{4} = 4.5mA$$

$$V_{oc2} \cong \frac{kT}{q} \ln\left(\frac{6 J_2}{I_0}\right) \rightarrow V_{oc2} - V_{oc1} = \frac{kT}{q} \ln\left(\frac{J_2}{J_1}\right)$$

$$\rightarrow V_{oc2} = V_{oc1} + \frac{kT}{q} \ln\left(0.25\right) = 0.394\,V$$

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c) Ricavare il punto di lavoro ottimo e calcolare FF

La cella eroga una potenza $P = IV = -I_{PH} V + I_0 V\left(e^{qV/kT} - 1\right)$

Per trovare il punto di lavoro ottimo si pone $\frac{dP}{dV} = 0$

$$\frac{dP}{dV} = -I_{PH} + I_0\left(e^{qV/kT} - 1\right) + I_0 V e^{qV/kT} \cdot \frac{q}{kT} = 0; \quad \frac{kT}{q} = V_{th}$$

$$= -I_{PH} + \left(I_0 + \frac{I_0 V}{V_{th}}\right) e^{qV/kT} - I_0 = 0$$

$$\rightarrow e^{\frac{V_m}{V_{th}}} = \frac{I_{PH} + I_0}{I_0\left(1 + \frac{V_m}{V_{th}}\right)}$$

$$V_m = V_{th} \ln\left(\frac{I_{PH} + I_0}{I_0\left(1 + \frac{V_m}{V_{th}}\right)}\right)$$

Soluzione iterativa:

$V_m^{(0)} = 0.4\,V$ guess iniziale

$V_m^{(1)} = 0.357\,V$

$V_m^{(2)} = 0.3601\,V$

$V_m^{(3)} \cong 0.36\,V$

$\Rightarrow$ $V_m = 0.36\,V$

$I_m = -16.8\,mA$

$$FF = \frac{I_m V_m}{I_{sc} V_{oc}} = 78.1\%$$