

# SOLUZIONI APPELLO 21-7-2020

1) Neutroni  $\rightarrow$  3 GRADI LIBERTÀ

$$E_s = \frac{3}{2} kT$$

$$\bullet E_c = \frac{1}{2} m_n N^2 = \frac{1}{2} m_n \frac{h^2}{m_n^2 \lambda^2} = \frac{1}{2} \frac{h^2}{m_n \lambda^2} \Rightarrow \lambda^2 = \frac{h^2}{2 m_n E_c}$$

$$\lambda = \frac{h}{\sqrt{2 m_n E_c}} = m_n \nu$$

$$\bullet E_c = E_s \Rightarrow \lambda^2 = \frac{h^2}{2 m_n \frac{3}{2} kT} \Rightarrow \lambda = \frac{h}{\sqrt{3 m_n kT}}$$

$$= 0,107 \text{ nm}$$

= RETICOLO CUBICO SCUPICE, DIFERENZE  
ALLA BANDA

$$2d \sin \theta = m\lambda, d = \frac{\lambda}{\sqrt{1^2 + 0^2 + 1^2}} = 0,355 \text{ nm}$$

$$\sin \theta = \frac{m\lambda}{2d} \Rightarrow \theta = \arcsin \left( \frac{m\lambda}{2d} \right)$$

$$\theta_{(m=1)} = 8,7^\circ$$

$$\theta_{(m=2)} = 17,6^\circ$$

$$\theta_{(m=3)} = 27^\circ$$

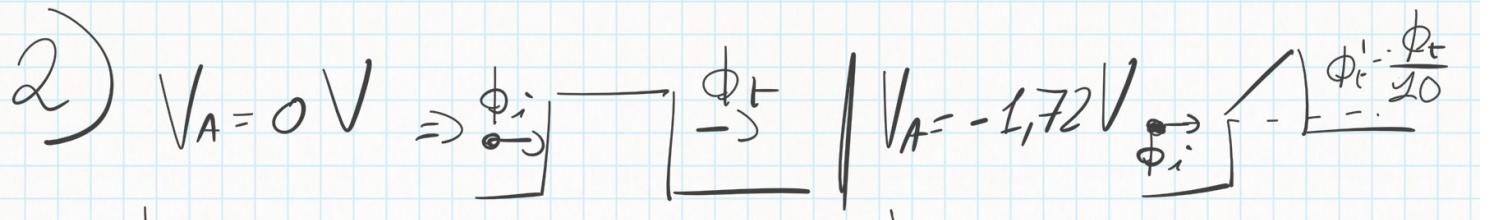
$$\theta_{(m=4)} = 37,2^\circ$$

$$\theta_{(m=5)} = 59^\circ$$

$$\theta_{(m=6)} = 65^\circ$$

$$\theta_{(m=7)} = \beta$$

\* CI SONO 6 PICCHI  
IN TOTALE



$$\bullet T_1 = e^{-2\sqrt{2m(V_0-E)}\alpha} \quad \bullet T_2 = e^{-2\int_0^a \sqrt{2m(V_0+9Fx-E)} dx}$$

$$\bullet T_2 = e^{-\frac{2\sqrt{2m}}{\hbar} \int_{V_0-E}^{V_0+9F\alpha-E} (\sqrt{z}) \frac{dz}{9F}}$$

$$z = V_0 + 9Fx - E; \quad dz = 9Fdx$$

$$T_2 = e^{-\frac{2}{3} \frac{\sqrt{2m}}{9V_0\hbar} a \left[ (V_0+9V_1-E)^{3/2} - (V_0-E)^{3/2} \right]}$$

$$|\text{Field}| = |\vec{F}| = |\nabla V| \Rightarrow \text{oppone RICONDA CITE} \quad \vec{F} = -\vec{\nabla} V$$

$$\bullet \frac{T_2}{T_1} = \frac{1}{10} = \frac{e^{-\frac{2}{3} \frac{\sqrt{2m}}{9V_1\hbar} a \left[ (V_0+9V_1-E)^{3/2} - (V_0-E)^{3/2} \right]}}{e^{-\frac{2\sqrt{2m}(V_0-E)}{\hbar} a}}$$

$$= \frac{1}{10} = e^{-a(\lambda - \beta)}$$

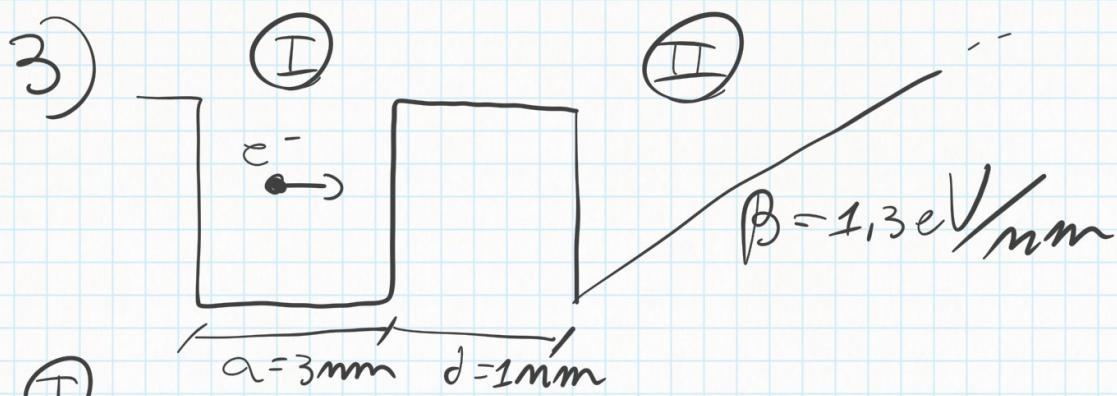
$$\lambda = 1,53 \cdot 10^{10} \text{ m}^{-1}$$

$$\beta = 1,21 \cdot 10^{10} \text{ m}^{-1}$$

$$\bullet \ln 10 = a(\lambda - \beta) \Rightarrow a = \frac{\ln(10)}{\lambda - \beta}$$

$$\bullet a = 7,2 \text{ \AA} = 0,72 \text{ mm}$$

$$\bullet \lambda \cdot \beta = 8,712, \quad \alpha = 11,010 \gg 1 \quad \left. \begin{array}{l} \text{APPROSSIM.} \\ \text{VALIDA} \\ \text{PER} \\ T_1 \text{ e } T_2 \end{array} \right\}$$



①

- APPNAX BUCA PARETI  $E_m = \frac{\hbar^2}{8ma^2} m^2 \Big|_{m=1} = 51,8 \text{ meV} (E_0)$

- $\langle E \rangle = \frac{t_{air}}{n}$ , con  $t_{air} = \frac{2a}{n}$ ,  $n = \sqrt{\frac{2G_1}{m}}$   
 $\rightarrow n = e^{-\frac{2\sqrt{2m(b-E)}}{\hbar}}$

$$\langle E \rangle = \frac{595 \text{ fs}}{51535 \cdot 10^{-5}} = 1,116 \text{ ms}$$

- ② Per trovare gli autovalori della buca rettilinea il primo per la indeterminazione di Heisenberg.

- $\Delta x \cdot \Delta p \approx m\hbar$
- $E = \beta \times \Rightarrow \Delta x = \frac{E}{\beta}$
- $G = \frac{\hbar^2}{2m} \Rightarrow \Delta p = \sqrt{2mE}$
- $E_n = \left( \frac{\beta}{\sqrt{2m}} + n \right)^{2/3} \quad \begin{cases} \Delta x \cdot \Delta p = \frac{E^{3/2}}{\beta} \sqrt{2m} \approx m\hbar \\ \rightarrow n=1 \Rightarrow E_{n=1} = 0,500 \text{ eV} \\ \rightarrow n=2 \Rightarrow E_{n=2} = 0,636 \text{ eV} \\ \rightarrow n=3 \Rightarrow \dots \end{cases}$

- Essendo  $E_{\frac{I}{II}} \approx 10 E_0$  NON ESISTE UN AUTOVALORE DELLA BUCA ② UNICO, A MENO DEL 5% ALLA AUTOVALORE DELLA BUCA ①

$$5) N_g = \frac{\partial \omega}{\partial k} \Big|_{k_0} = \frac{1}{\hbar} \frac{\partial E}{\partial k} \Big|_{k_0}$$

$$\sigma_x = \sqrt{\frac{\alpha^2 + \beta^2 t^2}{2}}, \text{ com } \begin{aligned} \beta &= \frac{1}{2} \frac{\partial^2 \omega}{\partial k^2} \Big|_{k_0} - \frac{1}{2\hbar} \frac{\partial^2 E}{\partial k^2} \Big|_k \\ \alpha &= \frac{1}{2\hbar k} = 5,55 \cdot 10^{-16} \text{ m}^2 \end{aligned}$$

- $E(k) = E_0 + E_0 \sin\left(\frac{2}{3}ka\right)$
- $\frac{\partial E(k)}{\partial k} = \frac{2}{3}aE_0 \cos\left(\frac{2}{3}ka\right)$
- $\frac{\partial^2 E(k)}{\partial k^2} = -\frac{4}{9}a^2E_0 \sin\left(\frac{2}{3}ka\right)$

•  $k_0 = \frac{3\pi}{4a}$ ;  $\sigma_k = 3 \cdot 10^7 \text{ m}^{-1}$ ;  $t = 2 \text{ ms}$ ;  $E_0 = 1,78 \text{ eV}$

$a = 0,6 \text{ nm};$

$$\Rightarrow N_g = \frac{E_0}{\hbar} \frac{2}{3} a \cos\left(\frac{2}{3} \frac{3\pi}{4a} a\right) = 0 \frac{\text{m}}{\text{s}}$$

$$\Rightarrow N_{\text{fase}} = \frac{\omega}{k} \Big|_{k_0} = \frac{E(k)}{\hbar k} \Big|_{k_0} = \frac{E_0 + E_0 \sin\left(\frac{2}{3} ka\right)}{\hbar k_0} = 1,3 \cdot 10^6 \frac{\text{m}}{\text{s}}$$

$$\Rightarrow \beta = \frac{1}{2\hbar} \cdot \left( -\frac{4}{9}a^2 E_0 \sin\left(\frac{2}{3} \frac{3\pi}{4a} a\right) \right) = -\frac{2}{9} \frac{a^2 E_0}{\hbar}$$

$$= -2,16 \cdot 10^{-4} \frac{\text{m}^2}{\text{s}}$$

$$\Rightarrow \sigma_x(t=2 \text{ ms}) = \sqrt{\frac{\alpha^2 + \beta^2 t^2}{2}} = 2,356 \cdot 10^{-3} \text{ m}$$

5) Elettrone libero:  $\psi(x) \propto e^{i(kx - \omega t)}$

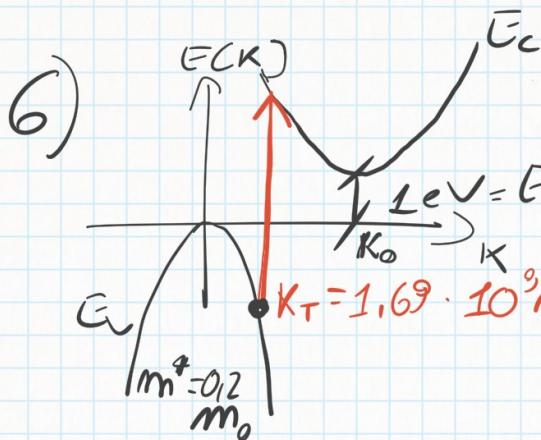
$$\hat{O}\psi = d\psi$$

↑ OPERATORE  
 ↓ AUTOVALORE  
 AUTOFUNZIONE

$$\hat{O} = \frac{1}{2} \frac{\partial^2}{\partial x^2}$$

$$\frac{1}{2} \frac{\partial^2}{\partial x^2} [A e^{i(kx - \omega t)}] = \underbrace{(ik)(-i\omega)}_{\text{AUTOVALORE}} A e^{i(kx - \omega t)}$$

$$d = +\frac{k\omega}{2} = \frac{\sqrt{2mE}}{25} \frac{E}{\hbar} = 1,5 \cdot 10^{25} \frac{1}{\text{ms}}$$



- $E_c(k) = E_{GAP} + B(k - k_0)^2$
- $E_v(k) = -Ak^2$
- $\frac{\partial}{\partial k} [E_c(k) - E_v(k)] = 0$
- $2B(k - k_0) + 2Ak = 0$

$$\bullet A = \frac{\hbar^2}{2m_h^*} = 1,907 \cdot 10^{-19} \text{ eVm}^2 \rightarrow k_T = \frac{Bk_0}{A+B}$$

$$\bullet B = \frac{Ak_T}{k_0 - k_T} = 9,76 \cdot 10^{-19} \text{ eVm}^2$$

$$\Rightarrow B = \frac{\hbar^2}{2m_c^*} \rightarrow m_c^* = \frac{\hbar^2}{2B} = 0,04 m_0$$

$$\Rightarrow N_{e^-} = \frac{1}{\hbar} \frac{\partial E_c(k)}{\partial k} = \frac{2B(k_T - k_0)}{\hbar} = -1,215 \cdot 10^6 \frac{\text{m}}{\text{s}}$$

$$\Rightarrow \lambda_{\text{Fotonen}} = \frac{\hbar c}{E_g + Ak_i^2 + B(k_T - k_0)^2} = 1,753 \text{ nm}$$

$$\Rightarrow \Delta E_{\text{crossover}} = B(k - k_0)^2 = 165 \text{ meV}$$

$$\Rightarrow \bar{N}_{\text{Fotonen}} = \frac{\Delta E}{E_{\text{nhm}}} \approx 7$$

$$\Rightarrow \Delta k = (k_T - k_0) \Rightarrow \bar{k}_{\text{phonon}} = \frac{\Delta k}{N_{\text{Fotonen}}} = 5,86 \cdot 10^6 \text{ m}^{-1}$$

$$\Rightarrow \left. \begin{array}{l} n = N_c \cdot e^{-\frac{E_c - E_F}{kT}} \\ p = N_V e^{-\frac{E_F - E_V}{kT}} \end{array} \right\} np = n_i^2$$

$$n_i^2 = N_c N_V e^{-\frac{E_G}{kT}} \Rightarrow E_G = -2kT \ln \frac{n_i}{\sqrt{N_c N_V}} = 5,5eV$$

$$8) n_i = \sqrt{N_c N_v} e^{-\frac{E_G}{2kT}}$$

$$N_c = \frac{n_i^2}{N_v} e^{\frac{E_G}{kT}} = 8,8 \cdot 10^{16} \text{ cm}^{-3}$$

$$N_c = \frac{1}{5\hbar^3} \left( \frac{2m^* kT}{\pi} \right)^{3/2}$$

$$m^* = \frac{\pi}{2kT} (5\hbar^3 N_c)^{2/3} = 0,023 m_0$$

$$9) \boxed{3D} \bullet m = \int_{E_{Co}}^{\infty} g(E) f(E) dE$$

$$\Rightarrow m = \int_{E_{Co}}^{E_F} g(E) f(E) dE$$

APPROX. METALLO

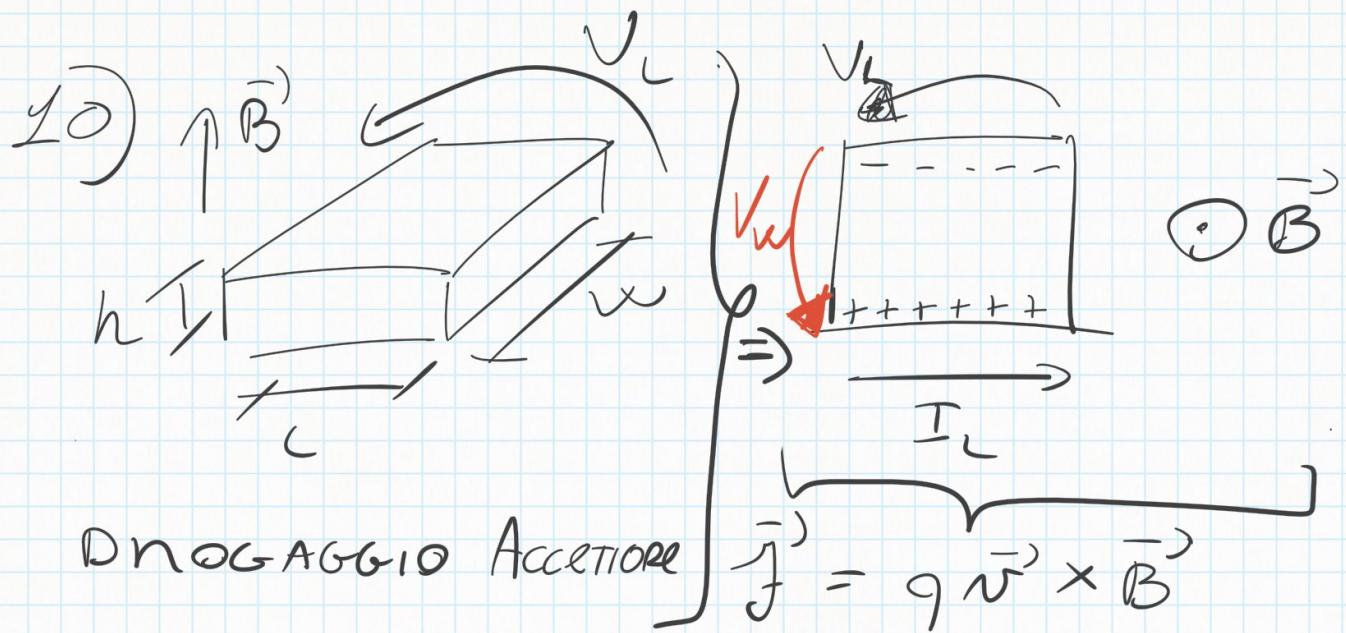
$$m = \frac{(2m^*)^{3/2}}{2\pi^2 h^3} \int \sqrt{E_F - E} dE = \frac{(2m_n^*)^{3/2}}{3\pi^2 h^3} (E_F - E_{Co})^{3/2}$$

$$= 5,18 \cdot 10^{22} \text{ cm}^{-3}$$

$$\bullet C(E) = \frac{1}{m} \int_{E_{Co}}^{\infty} E g(E) f(E) dE =$$

$$\approx \frac{1}{\frac{(2m^*)^{3/2}}{3\pi^2 h^3} (E_F)^{3/2}} \cdot \frac{(2m^*)^{3/2}}{2\pi^2 h^3} (E_F)^{5/2} =$$

$$= \frac{3}{5} \quad E_F = 2,4 \text{ eV}$$



- $I = q n v A = q n v_F A - q n v_F \frac{V_L}{L} h \omega$

- $\mu_p = \frac{I_L}{V_L q n h \omega} = 1132 \frac{\text{cm}^2}{\text{Vs}}$

$$n \approx N_A$$

- $|j_L| = q v \mathcal{B} = q \mu_p \frac{V_L}{L} \mathcal{B} = q F_\omega = q \frac{V_\omega}{\omega}$

- $V_\omega = \frac{W \mu_p V_L \mathcal{B}}{L} = 65 \text{ mV}$