

ES1

Esperimento di diffrazione a raggi X avente come target un cristallo con reticolo cubico semplice

$$\theta_1^{(100)} = 38^\circ$$

- i) $\theta_1^{(110)}$
- ii) $\theta_1^{(111)}$

Condizione di interazione costruttiva di Bragg nel caso del primo ordine di diffrazione ($m=1$) $\Rightarrow 2d \sin \theta_1 = \lambda$

dove $d = \begin{cases} a & \text{se si considerano i piani } (100) \\ \frac{a}{\sqrt{2}} & \text{se si considerano i piani } (110) \\ \frac{a}{\sqrt{3}} & \text{se si considerano i piani } (111) \end{cases}$

$$\begin{array}{l} ① \\ ② \\ ③ \end{array} \left\{ \begin{array}{l} 2a \sin \theta_1^{(100)} = \lambda \\ 2 \frac{a}{\sqrt{2}} \sin \theta_1^{(110)} = \lambda \\ 2 \frac{a}{\sqrt{3}} \sin \theta_1^{(111)} = \lambda \end{array} \right.$$

$$\frac{②}{①} \Rightarrow \sin(\theta_1^{(110)}) = \sqrt{2} \sin \theta_1^{(100)} \Rightarrow \theta_1^{(110)} = 60,53^\circ$$

$$\frac{③}{①} \Rightarrow \sin(\theta_1^{(111)}) = \sqrt{3} \sin \theta_1^{(100)} > 1 \Rightarrow \text{non si osserva alcun picco di diffrazione se si considerano i piani } (111)$$

ES2

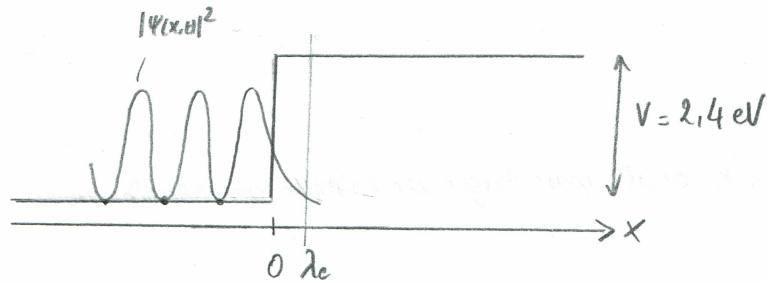
$$\hat{p} = -i\hbar \frac{d}{dx}$$

$$\hat{x}^2 = x^2$$

$$[x^2 p] \psi(x,t) = -x^2 i\hbar \frac{d}{dx} \psi(x,t) - \left(-i\hbar \frac{d}{dx} (\psi(x,t) \cdot x^2) \right) = -i\hbar x^2 \frac{d\psi(x,t)}{dx} + i\hbar \frac{d\psi(x,t)}{dx} x^2 + 2i\hbar x \psi(x,t)$$

$$\Rightarrow [x^2 p] = 2i\hbar x \neq 0 \quad \forall \psi(x,t) \quad \Rightarrow \hat{x}^2 e \hat{p} \text{ non commutano}$$

ES3



i) $\frac{P(x=\lambda_e)}{P(x=0)}$

$$E = \frac{\hbar^2}{2m\lambda_e^2} \Rightarrow \lambda_e = \frac{\hbar}{\sqrt{2mE}} = 8,47 \text{ \AA}$$

$$x > 0 : \psi(x) = D e^{-\alpha x} \Rightarrow P = |\psi(x)|^2 = D^2 e^{-2\alpha x} \text{ con } \alpha = \frac{\sqrt{2m(V-E)}}{\hbar} \approx 2,8 \cdot 10^9 \text{ m}^{-1}$$

$$\frac{P(x=\lambda_e)}{P(x=0)} = \frac{D^2 e^{-2\alpha\lambda_e}}{D^2} = e^{-2\alpha\lambda_e} = 8,65 \cdot 10^{-3}$$

ES4

Buca armonica

$$V(x) = \frac{1}{2} \alpha x^2$$

$$\alpha = 0,5 \text{ N m}^{-1}$$

i) E_0

ii) $\lambda_{\text{photou}}, E_1 - E_0$

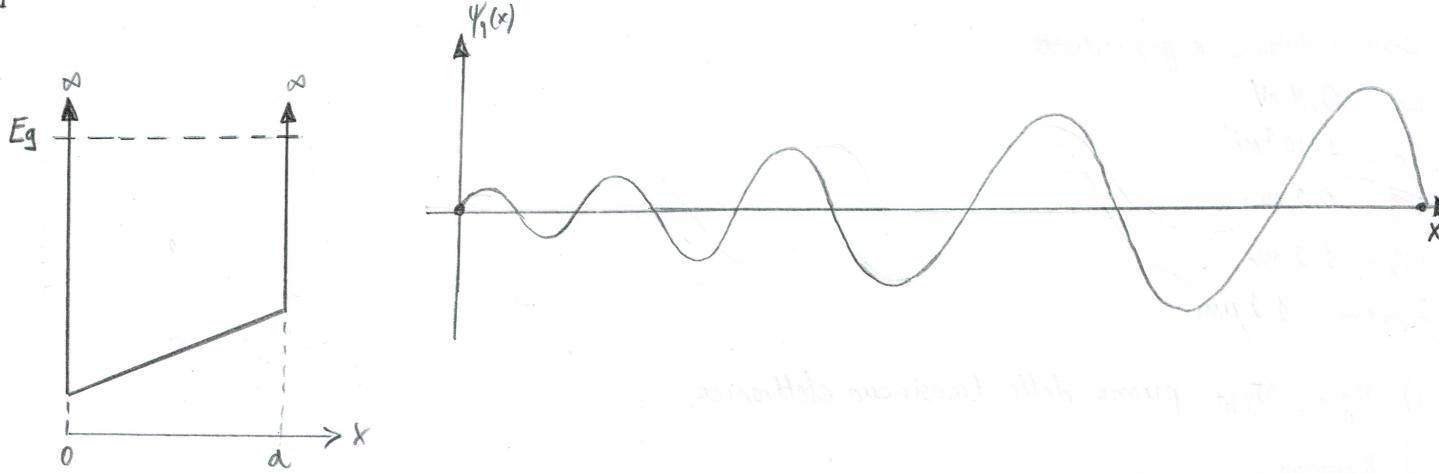
iii) $\lambda_{\text{photou}}, E_4 - E_3$

i) $E_m = \hbar \omega_0 \left(M + \frac{1}{2} \right) \rightarrow E_0 = \frac{\hbar \omega_0}{2} = \frac{\hbar}{2} \sqrt{\frac{\alpha}{m}} = 0,244 \text{ eV}$

ii) $\Delta E_{1,0} = \hbar \omega_0 \rightarrow \lambda = \frac{hc}{\hbar \omega_0} = \frac{c \cdot 2\pi}{\omega_0} = 2,54 \mu\text{m}$

iii) $\Delta E_{3-2} = \Delta E_{1,0} = \hbar \omega_0 \rightarrow \lambda_{3-2} = \lambda_{1-0} \Rightarrow$ livelli energetici delle buce quadrotice sono spaziati uniformemente

ES5

 $\Psi_1(x) \rightarrow \text{Gausse}$

$$V = \infty \Rightarrow \Psi_1(0) = \Psi_1(a) = 0$$

Da $x=0$ a $x=a$, V aumenta, $E_{\text{cin}} = E - V$ diminuisce \Rightarrow i) λ deve aumentare ($E_{\text{cin}} \propto \frac{1}{\lambda^2}$) e l'ampiezza dell'autofunzione (il cui modulo quadrato descrive le probabilità di trovare l'elettrone in un certo pto x) deve aumentare in quanto se $E_{\text{cin}} \downarrow \rightarrow v \downarrow \rightarrow$ probabilità di trovare l'elettrone aumenta.

ES6

$$E_c(k) = E_0 - E_1 \cos(ka) \quad \text{dove } E_0 = 1.5 \text{ eV} \text{ e } E_1 = 0.5 \text{ eV}$$

$$M^*(0) = 0,995 \text{ m}_0$$

$$V = 1 \text{ V}$$

$$T_{\text{Bloch}} = 10,35 \text{ ps}$$

i) N etii) ΔK in eneure di scattering dopo $\Delta t = 1 \text{ ps}$

$$\text{i) } M^*(0) = \frac{\hbar^2}{E_1 a^2} \rightarrow a = \frac{\hbar}{\sqrt{M^*(0) \cdot E_1}} = 4 \text{ \AA}$$

$$T_B = \frac{\hbar}{qFa} \rightarrow F = \frac{h}{qT_B a} \cong 10 \text{ KV/m}$$

$$F = \frac{V}{Na} \rightarrow N = \frac{V}{Fa} = 2500$$

$$\text{ii) In eneure di scattering, } \frac{dN}{dt} = \frac{qF}{\hbar} \rightarrow \Delta K = \frac{qF \Delta t}{\hbar} \rightarrow \Delta K = 1.5 \cdot 10^9 \text{ m}^{-1}$$

ES7

Semiconduttore a gap indiretto

$$E_g = 0,9 \text{ eV}$$

$$K_0 = 4 \cdot 10^9 \text{ m}^{-1}$$

$$M_e^* = 0.2 \text{ Mo}$$

$$M_h^* = 1.2 \text{ Mo}$$

$$\lambda_{\text{photon}} = 1.2 \mu\text{m}$$

i) $v_{g e^-}, v_{gh^+}$ piume delle transizioni elettroniche

ii) K_{phonon}

$$\text{i)} E_c(u) = E_g + B(u - K_0)^2 \quad \text{con} \quad B = \frac{\hbar^2}{2m_e^*} = 1.91 \cdot 10^{-19} \text{ eV} \cdot \text{m}^2$$

$$E_v(u) = -AK^2 \quad \text{con} \quad A = \frac{\hbar^2}{2m_h^*} = 3.18 \cdot 10^{-20} \text{ eV} \cdot \text{m}^2$$

$$\frac{hc}{\lambda} > E_{\text{gap}} \Rightarrow E_{\text{ph}} = E_g + \Delta E_v \rightarrow \Delta E_v = \frac{hc}{\lambda} - E_g = AK^2 \Rightarrow K_{\text{fin}} = \sqrt{\frac{\Delta E_v}{A}} = 2.06 \cdot 10^9 \text{ m}^{-1}$$

$$v_{g e^-}(K_0) = 0$$

$$v_{gh^+}(K_{\text{fin}}) = \frac{1}{\hbar} \left. \frac{dE_v}{dK} \right|_{K_{\text{fin}}} = - \frac{2AK_{\text{fin}}}{\hbar} \approx -2 \cdot 10^5 \text{ m/s}$$

ii) Conservazione momento reticolare $\Rightarrow K_{\text{phonon}} = |K_{\text{fin}} - K_0| = +1.94 \cdot 10^9 \text{ m}^{-1}$

ES8

Semiconduttore si intusco ($M_e^* = 0.26 \text{ Mo}$)

$$T_{\text{m}} = 10^{-14} \text{ s}$$

$$T_1 = 150 \text{ K}$$

i) $\lambda(T_1)$

ii) $\lambda(T_2 = 450 \text{ K})$

$$\text{i)} V_{\text{m}} = \sqrt{\frac{3kT_1}{M_e^*}} = 1.62 \cdot 10^5 \text{ m/s}$$

$$\lambda(T_1) = V_{\text{m}} \cdot T_{\text{m}} = 1.62 \text{ nm}$$

$$\text{ii)} \lambda = V_{\text{m}} T_{\text{m}} \propto \sqrt{T} \cdot T^{-3/2} \propto T^{-1} \Rightarrow \frac{\lambda(T_2)}{\lambda(T_1)} \sim \frac{T_1}{T_2} = \frac{150}{450} = \frac{1}{3} \quad (T \uparrow \rightarrow \text{oriente debole fononi} \rightarrow \lambda \downarrow)$$

ES9

Semiconduttore drogato p GeAs

$$N_A = 10^{17} \text{ cm}^{-3}$$

$$T = 200 \text{ K}$$

$$E_{\text{GAP}} = 1.424 \text{ eV}$$

$$M_e^* = 0,067 \text{ Mo}$$

$$M_{lh}^* = 0,082 \cdot \text{Mo}$$

$$M_{hh}^* = 0,51 \text{ Mo}$$

i) disegnare il diagramma a bande del semiconduttore

$$M_{\text{DOS},m}^* = M_e^* = 0,067 \text{ Mo} \quad (\text{minimo isotopo in } \Gamma \rightarrow g=1)$$

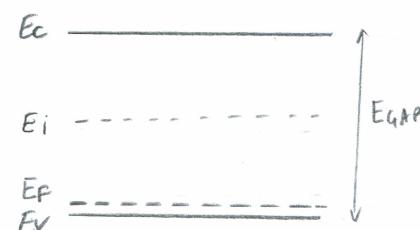
$$M_{\text{DOS},p}^* = (M_{lh}^{*3/2} + M_{hh}^{*3/2})^{2/3} \approx 0,53 \text{ Mo}$$

$$N_c = \frac{1}{4\pi^3} \left(\frac{2M_{\text{DOS},m}^* kT}{\pi} \right)^{3/2} = 2.36 \cdot 10^{17} \text{ cm}^{-3}$$

$$N_v = \frac{1}{4\pi^3} \left(\frac{2M_{\text{DOS},p}^* kT}{\pi} \right)^{3/2} = 5.284 \cdot 10^{18} \text{ cm}^{-3}$$

$$p = N_v e^{\frac{E_F - E_v}{kT}} \xrightarrow{p \sim N_A \text{ a } T=200 \text{ K}} E_F = E_v + kT \log\left(\frac{N_v}{N_A}\right) \approx E_v + 68,4 \text{ meV} \quad (\text{Approx MB è valida in quanto } 3kT < 68,4 \text{ meV})$$

$$E_i = \frac{E_{\text{GAP}}}{2} + \frac{kT}{2} \log\left(\frac{N_v}{N_c}\right) \approx 0,74 \text{ eV}$$



ES10

Semiconduttore drogato n Ge

$$N_D = 10^{16} \text{ cm}^{-3}$$

$$P_{ni}(300K) = 3 \cdot 10^{-3}$$

i) Energia di ionizzazione ($E_c - E_D$)

$T = 300 \text{ K} \rightarrow$ ionizzazione quasi completa $m \sim N_D$ ($N_D < N_c$)

$$P_{ni} = \frac{N_{D0}}{N_{D0} + m} = \frac{1}{1 + \frac{N_c}{2N_D} e^{-\frac{E_c - E_D}{kT}}} \xrightarrow{N_D \ll N_c} P_{ni} \sim \frac{2N_D}{N_c} e^{\frac{E_c - E_D}{kT}} \xrightarrow{} E_c - E_D = kT \log\left(\frac{N_c \cdot P_{ni}}{2N_D}\right) = 11.5 \text{ meV}$$