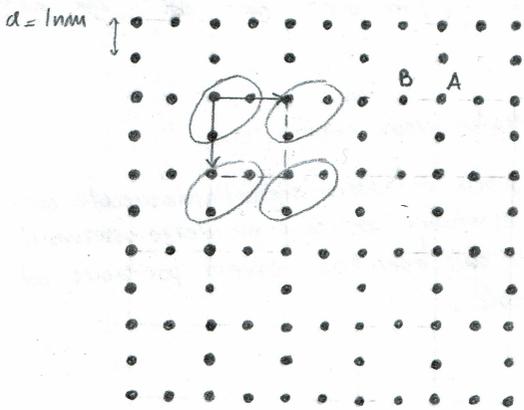


ES1



1) IL CRISTALLO 2D NON E' UN RETICOLO DI BRAVAIS
 → DAL PTO A NON VEDO ESATTAMENTE QUELLO CHE SI VEDE DAL PTO B

2) CRISTALLO; RETICOLO QUADRATO DI LATO $2a$
 + BASE COMPOSTA DA 3 ATOMI

$$\rho_s = \frac{N_{\text{at}} \text{ celle u}}{\text{Area celle u}} = \frac{3}{(2a)^2} = \frac{3}{4a^2} = 7.5 \cdot 10^{13} \text{ cm}^{-2}$$

In questo caso considero celle primitive

ES2

Sfere in tungsteno

$d = 2 \text{ cm}$

$T_{\text{sfera}} = 2000^\circ\text{C} \text{ (2273 K)}$

$P_{\text{sfera}} = 40\% P_{\text{BB}}|_{d, T_{\text{sfera}}}$

i) $\lambda_{\text{peak, BB2}}|_{d, P_{\text{sfera}}}$

$P_{\text{sfera}} = 0.4 \sigma T_{\text{sfera}}^4 A_{\text{sfera}} = 0.4 \sigma T_{\text{sfera}}^4 4\pi \left(\frac{d}{2}\right)^2 = 0.4 \sigma T_{\text{sfera}}^4 \pi d^2 \sim 761 \text{ W}$

$P_{\text{BB2}} = \sigma T_{\text{BB2}}^4 \pi d^2 = P_{\text{sfera}} \rightarrow T_{\text{BB2}} = \sqrt[4]{\frac{P_{\text{sfera}}}{\sigma \pi d^2}} \sim 1808 \text{ K}$

→ Legge Wien $\lambda_{\text{peak, BB2}} \cdot T_{\text{BB2}} = C_w$

↳ $\lambda_{\text{peak, BB2}} = \frac{C_w}{T_{\text{BB2}}} = 1,604 \mu\text{m}$

ES3

Effetto fotoelettrico

$K = 3 \text{ W}$

i) λ' t.c. $K' = 2K$

$E_{ph} = K + W$

$\begin{cases} h\nu = K + W = 3W + W = 4W \\ h\nu' = K' + W = 2K + W = 6W + W = 7W \end{cases}$

$\frac{h\nu}{h\nu'} = \frac{\lambda'}{\lambda} = \frac{4W}{7W} = \frac{4}{7} \Rightarrow \lambda' = \frac{4}{7} \lambda$

ES4

$$[x, H] = (\hat{x} \hat{H} - \hat{H} \hat{x}) \psi(x,t) = -\frac{\hbar^2}{2m} x \frac{d^2 \psi(x,t)}{dx^2} + x V(x,t) \psi(x,t) + \frac{\hbar^2}{2m} \frac{d^2}{dx^2} (x \cdot \psi(x,t)) - x V(x,t) \psi(x,t) =$$

$$= -\frac{\hbar^2}{2m} x \frac{d^2 \psi(x,t)}{dx^2} + \frac{\hbar^2}{2m} \frac{d}{dx} (\psi(x,t) + x \frac{d\psi(x,t)}{dx}) = -\frac{\hbar^2}{2m} x \frac{d^2 \psi(x,t)}{dx^2} + \frac{\hbar^2}{2m} \frac{d\psi(x,t)}{dx} + \frac{\hbar^2}{2m} \frac{d\psi(x,t)}{dx} + \frac{\hbar^2}{2m} \frac{d^2 \psi(x,t)}{dx^2} x =$$

$$= \cancel{\frac{\hbar^2}{2m} \frac{d^2 \psi(x,t)}{dx^2}} = \frac{\hbar^2}{m} \frac{d\psi(x,t)}{dx} = -i \hbar \frac{p}{m} \psi(x,t) \neq 0 \quad \forall \psi(x,t) \rightarrow \text{i due operatori non commutano}$$

$$\hat{p}_x = -i \hbar \frac{d}{dx}$$

$$\hat{p}_x \psi(x,t) = p \psi(x,t)$$

↳ non è possibile misurare simultaneamente con precisione arbitraria le due grandezze osservabili associate ai due operatori, ovvero posizione ed energia totale.

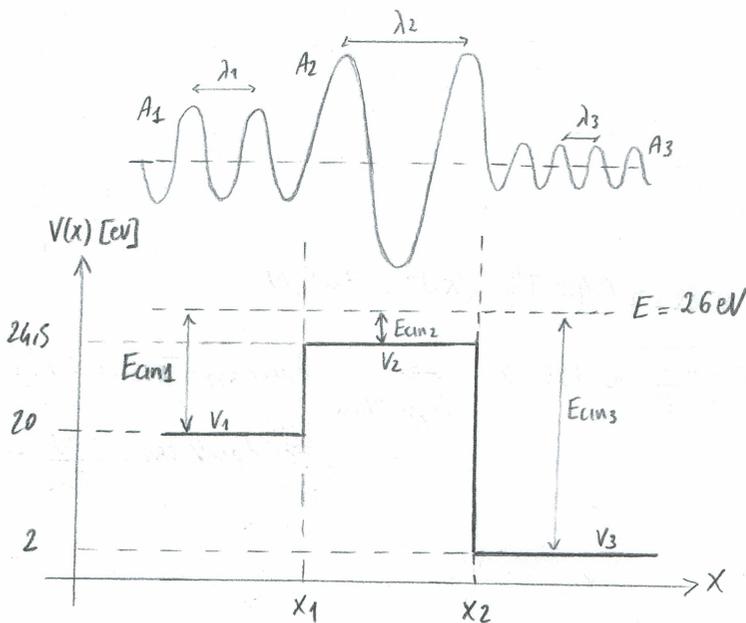
ES5

$E = 26 \text{ eV}$

$\lambda_2 = 1 \text{ mm}$

$\lambda_2 = 2 \lambda_1$

$\lambda_2 = 4 \lambda_3$



$$\lambda_2 = 1 \text{ mm} \rightarrow E_{cin2} = \frac{h^2}{2m\lambda_2^2} \approx 1,5 \text{ eV} \rightarrow V_2 = E - E_{cin2} = 24,5 \text{ eV}$$

$$\lambda_1 = \frac{\lambda_2}{2} \rightarrow E_{cin1} = 4 E_{cin2} = 6 \text{ eV} \Rightarrow V_1 = E - E_{cin1} = 20 \text{ eV}$$

$$\lambda_3 = \frac{\lambda_2}{4} \rightarrow E_{cin3} = 16 E_{cin2} = 24 \text{ eV} \rightarrow V_3 = E - E_{cin3} = 2 \text{ eV}$$

$$\left. \begin{array}{l} \\ \\ \end{array} \right\} V_2 - V_1 = \Delta V_{12} = 4,5 \text{ eV}$$

$$E_{cin} \propto \frac{1}{\lambda^2} \rightarrow \lambda_2 > \lambda_1 > \lambda_3 \rightarrow E_{cin2} < E_{cin1} < E_{cin3}$$

$$A_2 > A_1 > A_3 \rightarrow V_2 < V_1 < V_3 \rightarrow E_{cin2} < E_{cin1} < E_{cin3}$$

ES6

Bucca pareti infinite

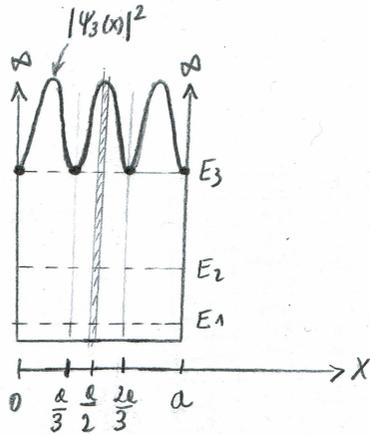
$$a = 5 \text{ nm}$$

$$\Delta x = 1 \text{ \AA}$$

i) E_1, E_2, E_3

ii) $x \in [0, a] + c \quad P=0$

iii) $P(x \in [\frac{a}{2} - \frac{\Delta x}{2}, \frac{a}{2} + \frac{\Delta x}{2}])$



i) Approx bucca pareti infinite

$$E_n = \frac{h^2 n^2}{8ma^2}$$

$$\hookrightarrow E_1 = \frac{h^2}{8ma^2} = 15,07 \text{ meV}$$

$$E_2 = \frac{h^2}{8ma^2} \cdot 4 = 60,3 \text{ meV}$$

$$E_3 = \frac{h^2}{8ma^2} \cdot 9 = 135,7 \text{ meV}$$

ii) $P=0 \rightarrow x=0, x=\frac{a}{3}, x=\frac{2a}{3}, x=a$

iii) $\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \rightarrow \psi_3(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{3\pi x}{a}\right)$

$$|\psi_3(x)|^2 = \frac{2}{a} \sin^2\left(\frac{3\pi x}{a}\right) \rightarrow P = \int_{\frac{a}{2} - \frac{\Delta x}{2}}^{\frac{a}{2} + \frac{\Delta x}{2}} \frac{2}{a} \sin^2\left(\frac{3\pi x}{a}\right) dx \approx \frac{2}{a} \cdot 1 \cdot \Delta x = \frac{2\Delta x}{a} = 4\%$$

$\sin^2\left(\frac{3\pi x}{a}\right) \sim 1 \quad x \sim \frac{a}{2}$

ES7

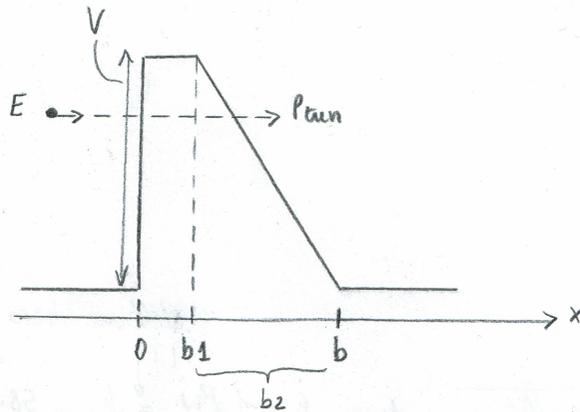
$$E = 2 \text{ eV}$$

$$V = 2,5 \text{ eV}$$

$$P_{\text{tun}} = 2,4 \cdot 10^{-3}$$

$$b_1 = 0,5 \text{ nm}$$

i) b



$$P_{\text{tun}} = P_{\text{tun}1} \cdot P_{\text{tun}2}$$

$$P_{\text{tun}1} = P_{\text{tun}}^{\text{wub}} = e^{-2 \frac{\sqrt{2m(V-E)}}{\hbar} b_1} = 2,68 \cdot 10^{-2}$$

$$P_{\text{tun}2} = \frac{P_{\text{tun}}}{P_{\text{tun}1}} = 8,96 \cdot 10^{-2} = e^{-\frac{4}{3} \frac{\sqrt{2m}}{\hbar} b_2 (V-E)^{3/2}} \rightarrow b_2 = -\frac{\log(P_{\text{tun}2})}{\frac{4}{3} \frac{\sqrt{2m}}{\hbar} (V-E)^{3/2}} = 2,5 \text{ nm}$$

$$b = b_1 + b_2 = 3 \text{ nm}$$

$$\psi(x,t) = a_1 \psi_1(x) e^{-i \frac{E_1 t}{\hbar}} + a_2 \psi_2(x) e^{-i \frac{E_2 t}{\hbar}} = e^{-i \frac{E_1 t}{\hbar}} (a_1 \psi_1(x) + a_2 \psi_2(x) e^{-i \frac{E_2 - E_1 t}{\hbar}}) =$$

$$= e^{-i \frac{E_1 t}{\hbar}} (a_1 \psi_1(x) + a_2 \psi_2(x) e^{-i \frac{\Delta E_{21} t}{\hbar}}) \quad \text{dove } a_1 \text{ e } a_2 \text{ t.c. } |a_1|^2 + |a_2|^2 = 1 \rightarrow a_1 = a_2 = \frac{1}{\sqrt{2}} \text{ ades.}$$

$$\nu_{osc} = \frac{\Delta E_{21}}{h} = 1.2 \text{ THz}$$

ES9

$$V(x,y) = \frac{1}{2} \alpha (x^2 + y^2)$$

$$\alpha = 0,0134 \text{ Nm}^{-1}$$

i) Energia e degenerazione del 7° autovalore

Potenziale armonico bidimensionale

$$\rightarrow x \rightarrow -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} + \frac{1}{2} \alpha x^2 \psi(x) = E_x \psi(x)$$

$$y \rightarrow -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(y)}{\partial y^2} + \frac{1}{2} \alpha y^2 \psi(y) = E_y \psi(y)$$

$$E = E_x + E_y = \hbar \omega_0 (m_x + \frac{1}{2}) + \hbar \omega_0 (m_y + \frac{1}{2}) = \hbar \omega_0 (m_x + m_y + 1)$$

$$\text{dove } \hbar \omega_0 = \hbar \sqrt{\frac{\alpha}{m}} \cong 80 \text{ meV}$$

$$E_7 = 7 \hbar \omega_0 \sim 560 \text{ meV}$$

$$g = 7 \text{ (7 stati di uguale energia)}$$

DEGENERAZIONE

m_x	m_y	E	g
0	0	$\hbar \omega_0$	1
0	1	$2 \hbar \omega_0$	2
1	0	$2 \hbar \omega_0$	2
1	1	$3 \hbar \omega_0$	3
2	0	$3 \hbar \omega_0$	3
0	2	$3 \hbar \omega_0$	3
1	2	$4 \hbar \omega_0$	4
2	1	$4 \hbar \omega_0$	4
3	0	$4 \hbar \omega_0$	4
0	3	$4 \hbar \omega_0$	4
2	2	$5 \hbar \omega_0$	5
1	3	$5 \hbar \omega_0$	5
3	1	$5 \hbar \omega_0$	5
0	4	$5 \hbar \omega_0$	5
4	0	$5 \hbar \omega_0$	5
4	1	$6 \hbar \omega_0$	6
1	4	$6 \hbar \omega_0$	6
3	2	$6 \hbar \omega_0$	6
2	3	$6 \hbar \omega_0$	6
5	0	$6 \hbar \omega_0$	6
0	5	$6 \hbar \omega_0$	6
1	5	$7 \hbar \omega_0$	7
5	1	$7 \hbar \omega_0$	7
2	4	$7 \hbar \omega_0$	7
4	2	$7 \hbar \omega_0$	7
3	3	$7 \hbar \omega_0$	7
6	0	$7 \hbar \omega_0$	7
0	6	$7 \hbar \omega_0$	7

E_7

ES10

Elettone libero

$$E = 25 \text{ eV}$$

$$\sigma_{x_0} = 1 \mu\text{m}$$

$$d = 1 \text{ m}$$

i) σ_x

ii) σ_x se $\sigma_{x_0} = 1 \mu\text{m} \rightarrow 10 \text{ nm}$

$$i) \sigma_x^2(t) = \alpha + \frac{\beta^2 t^2}{\alpha}$$

$$\sigma_x^2(t=0) = \alpha \rightarrow \sigma_{x_0} = \sqrt{\alpha}$$

$$\overset{\text{e-libero}}{E} = \frac{\hbar^2 k^2}{2m} \rightarrow \omega = \frac{E}{\hbar} = \frac{\hbar k^2}{2m} \rightarrow \beta = \frac{1}{2} \frac{d^2 \omega}{dk^2} = \frac{\hbar}{2m} \sim 58 \cdot 10^{-6} \frac{\text{m}^2}{\text{s}}$$

$$\bar{E} = \frac{d}{v} = \frac{d}{\sqrt{2E}} \cong 0,34 \mu\text{s}$$

$$\sigma_x = \sqrt{\alpha} \sqrt{1 + \left(\frac{\beta t}{\alpha}\right)^2} = \sigma_{x_0} \sqrt{1 + \left(\frac{\beta t}{\sigma_{x_0}^2}\right)^2} \rightarrow \sigma_x(\bar{E}) \cong 20 \mu\text{m}$$

ii) $\sigma_{x_0} = 10^{-8} \text{ m} \rightarrow \sigma_x(\bar{E}) = 2 \text{ mm} \rightarrow$ MINORE E' L'ALLARGAMENTO INIZIALE DEL PACCHETTO ELETTRONICO, MAGGIORE SARA' IL SUO ALLARGAMENTO FINALE