

[06/05/2013] Soluzioni

[Es 1]

A) NO → Areo non coperto
→ Non è una cella unitaria

Cella unitaria

N° atomi/cella = 1 → cella primitiva

Set vettori
primari

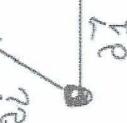


[Es 1]

Cella unitaria

N° atomi/cella = 1 → cella primitiva

Set vettori
primari

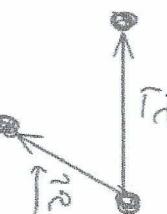


[Es 1]

Cella unitaria

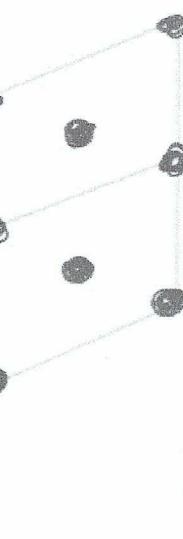
N° atomi/cella = 1 → cella primitiva

Set vettori
primari



E) N° atomi/cella = 2
→ Non è una cella unitaria

→ Non è una cella unitaria



[Es 1]

Cella unitaria

N° atomi/cella = 1

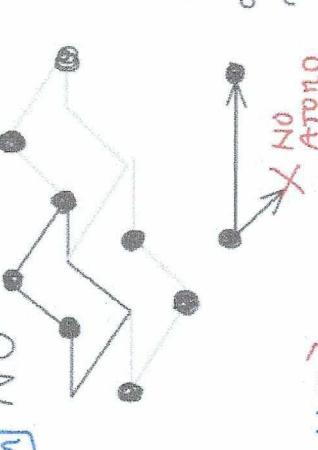
Set vettori
primari



Cella unitaria

N° atomi/cella = 1

Set vettori
primari



[Es 1]

Cella unitaria

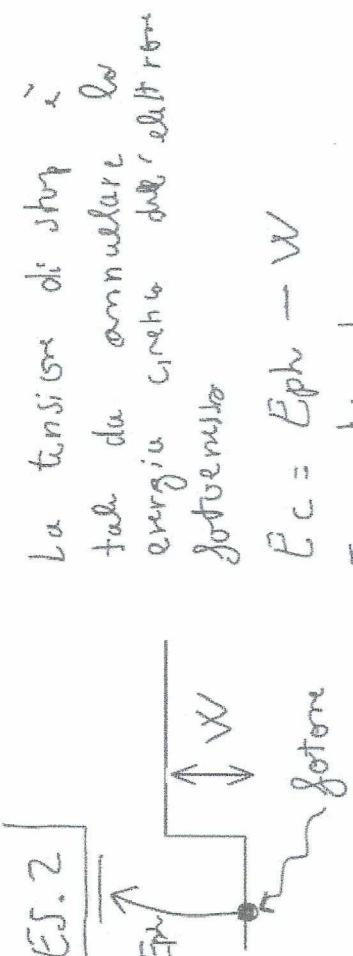
N° atomi/cella = 1

Set vettori
primari



27

Si noti che è possibile usare un'unica coppia di vettori primari per traslare tutta la cella unitaria primitiva.



La tensione di stop è
tale da annullare la
energia cinetica delle elettroni
gotenuti.

$$E_C = E_{ph} - W$$

$$E_C - q|V_{stop}| = 0$$

$$\rightarrow W = E_{ph} - q|V_{stop}| = \frac{hc}{\lambda} - q|V_{stop}| = 2,5 \text{ eV}$$

Per avere effetti fotovoltaici:

$$E_{ph} > W \rightarrow \frac{hc}{\lambda} > W \rightarrow \lambda \leq \frac{hc}{W} = \lambda_{max} = 500 \text{ nm}$$

$$\rightarrow \lambda_{min} = c/\gamma_{max} = 600 \text{ THz}$$

Nel caso $\lambda = 450 \text{ nm}$, $E_{ph} = 2,75 \text{ eV}$

$$\rightarrow E_C = E_{ph} - W = q|V_{stop}|$$

$$\rightarrow |V_{stop}| = 0,25 \text{ V}$$

$$+ \pi/2 \rightarrow n=1, \alpha=30^\circ \rightarrow \sin \alpha = 1/2$$

$$\rightarrow 2d \frac{1}{2} = 1 = 0,5 \text{ nm}$$

$$\rightarrow V_A = 6 \text{ V}$$

pioni (111) $\rightarrow d = \frac{2}{\sqrt{3}} = 1,15 \text{ nm}$

$$pioni (111) \rightarrow d = \frac{2}{\sqrt{3}} = 1,15 \text{ nm}$$

$$\rightarrow \sin \alpha = \frac{1}{2d} = \frac{\sqrt{3}}{2}, \alpha = 60^\circ$$

$E < V_0 \rightarrow$ tunneling due barrieria

$T = \exp(-2\sqrt{\frac{2m(V_0-E)}{\hbar}})$ $= 9,2 \cdot 10^{-8}$

$\phi_i = \phi_i \cdot T = 9,2 \cdot 10^{-11} \text{ eV} \cdot \text{s}^{-1}$

$E > V_0$
Risultato per T

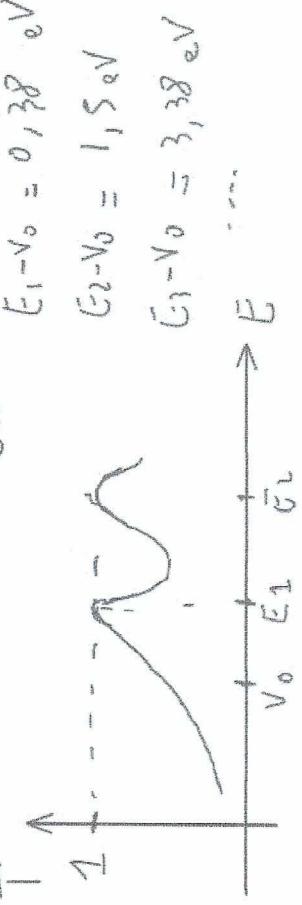
$$2K' \delta = 2\pi n$$

$$K'_n = \frac{\pi n}{\delta}$$

Es. 3 Energia cinetica elettronica fornita da elettroni

$$E_C = \frac{h^2}{2m\lambda^2} = qV_A$$

$$\rightarrow V_A = \frac{h^2}{2m\lambda^2 q}$$



$$E_1 - V_0 = 0,38 \text{ eV}$$

$$E_2 - V_0 = 1,5 \text{ eV}$$

$$E_3 - V_0 = 3,38 \text{ eV}$$

Risultati di diffusione per 2d sin α = n λ

pioni (100) $\rightarrow d = \vartheta = 0,5 \text{ nm}$

$$[Ex. 5] V(x) = \alpha x^\beta$$

Plasma imdeformazione in posizione per $\tilde{E} = \sqrt{\alpha}$

$$\alpha \Delta x^\beta = E \rightarrow \partial x = \left(\frac{E}{\alpha}\right)^{1/\beta}$$

Massimo deformazione in momento per $\tilde{E} = \tilde{E}_{\text{max}}$

$$\frac{\Delta p^2}{2m} = \tilde{E} \rightarrow \Delta p = \sqrt{2m\tilde{E}}$$

Principio della conservazione:

$$\Delta x \Delta p = n \hbar \quad \sqrt{2m\tilde{E}} = n \hbar \\ \left(\frac{E}{\alpha}\right)^{1/\beta} \cdot \sqrt{2m\tilde{E}} = n \hbar \\ \tilde{E}^{1/\beta + 1/2} = \frac{\alpha \sqrt{\beta}}{\sqrt{2m}} n \hbar$$

$$\rightarrow \tilde{E}_n = \left(\frac{\alpha}{\sqrt{2m}}\right)^{1/\beta} \cdot \left(n \hbar\right)^{\frac{2\beta}{\beta+2}}$$

$$\text{Soglia } \tilde{E}_n \propto n^{3/2} \rightarrow \frac{2\beta}{\beta+2} = \frac{3}{2}$$

$$2\beta = \frac{3}{2}\beta + 3 \rightarrow \boxed{\beta = 6}$$

$$[Ex. 6]$$

Approssimazione per σ con infiniti:

$$\tilde{E}_n = \frac{\hbar^2}{8m} n^2 \rightarrow \tilde{E}_1 = 0,5 \text{ eV} \quad \tilde{E}_2 = \sqrt{\frac{2\tilde{E}_1}{m}} = 4,56 \cdot 10^{-5} \text{ eV}$$

Soprattutto tunneling Fowler - Nordheim

$$t_{\text{tun}} = \frac{t_{2/3}}{\text{plan}} = \frac{\left(\frac{2.2}{2.5}\right)}{\exp\left[-\frac{4\sqrt{2m}}{q\hbar F} (V_0 - \tilde{E}_1)^{1/2}\right]} = 2.085$$

$$\rightarrow -\frac{4}{3} \frac{\sqrt{2m}}{q\hbar F} (V_0 - \tilde{E}_1)^{3/2} = \log \frac{e^2}{N t_{\text{tun}}}$$

$$\rightarrow F = -\frac{4}{3} \frac{\sqrt{2m}}{q\hbar} \frac{(V_0 - \tilde{E}_1)^{3/2}}{\log \frac{e^2}{N t_{\text{tun}}}} = 50 \frac{\text{MV}}{\text{cm}}$$

$$\underline{\text{Si vuole che } F \cdot b > V_0 - \tilde{E}_1 \rightarrow \text{trovare } F - N \text{ è costante}}$$

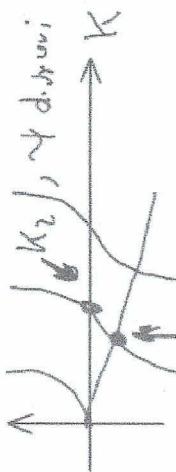
$$[Ex. 7] |\psi_{nr}|^2 = |\psi_1 + \psi_2|^2 \sim \cos(2\pi v t)$$

$$v = \frac{\tilde{E}_2 - \tilde{E}_1}{\hbar} \rightarrow \tilde{E}_2 - \tilde{E}_1 = \Delta E = \hbar v = 4.1 \text{ eV}$$

Soluzione: buona appross.

$$\tilde{E}_n = \frac{\hbar^2 K}{2m} \quad / \quad \tan(\hbar \vartheta/2) = 0, \text{ non par}$$

$$\tan(\hbar \vartheta/2) = -\frac{\hbar^2 K}{m \omega}, \text{ non pari}$$



$$K_2 = \frac{2\pi}{\lambda}, \quad \text{Psi}_{\text{d. sim.}} = K_2 \cdot 10^9 \text{ m}^{-1}$$

$$E_2 = \frac{\hbar^2 k_2}{2m} = 2,36 \text{ eV}$$

$$\rightarrow E_1 = E_2 - \Delta E = 2,31 \text{ eV} \quad , \quad K_1 = \sqrt{2m(E_1)} = 7,79 \cdot 10^7 \text{ m}^{-1}$$

$$\tan(k_1 \cdot 2r_2) = -\frac{\hbar k_2}{m \cdot m_0} \rightarrow m_0 = -\frac{\hbar k_1}{m k_{2n} (\mu_1 \mu_2)} = 2,37 \cdot 10^{-8} \text{ eV} \cdot \text{m}$$

$$[E_1 \cdot g] \text{ Minimo per } \frac{dU}{du} = 0$$

$$E_0 \sin(\mu_2) \left[2 \cos(\mu_2) - 1 \right] = 0$$

$$\left. \begin{array}{l} K=0 \\ K=\pm \frac{\pi}{2} \end{array} \right\} \text{ Caso limite del punto di equilibrio}$$

Dove sono le condizioni

$$\frac{dU}{du^2} = E_0 \cdot 2 \left[2 \cos^2(\mu_2) + \cos(\mu_2) - 2 \sin^2(\mu_2) \right] > 0$$

$$K=0 \quad \frac{dU}{du} = 3E_0 \cdot 2^2 > 0 \quad \text{H1H1H1}$$

$$K=\pm \frac{\pi}{3}, \quad \frac{dU}{du} = (1-\sqrt{3}) E_0 \cdot 2^2 < 0 \quad \text{H1H1H1}$$

$$K=\pm \frac{\pi}{2} \quad \frac{dU}{du} = E_0 \cdot 2^2 > 0 \quad \text{H1H1H1}$$

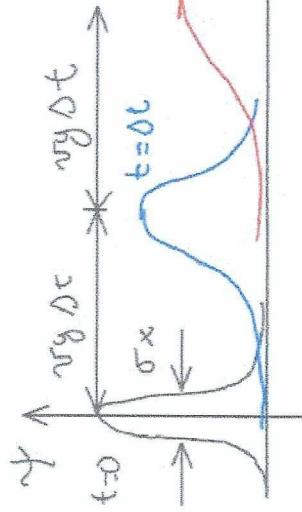
$$K=0 \quad m^* = \frac{\hbar^2}{\frac{d^2U}{du^2} \Big|_{u=0}} = \frac{\hbar^2}{3E_0 \cdot 2^2} = 0,102 \text{ m}^0$$

$$u=\frac{\pm \pi}{2} \quad m^* = \frac{\hbar^2}{E_0 \cdot 2^2} = 0,305 \text{ m}^0$$

E_2 è minimo dove m^* è più piccolo

$$\left. \begin{array}{l} \text{La moltiplicazione} \\ \text{quindi in } K=\pm \frac{\pi}{2} \end{array} \right\} \quad \begin{cases} \text{sg} = \frac{1}{h} \frac{dU}{du} \Big|_{u=u_1} > 0 \rightarrow \text{punto} \\ \text{verso} \end{cases}$$

$$\left. \begin{array}{l} \beta=\frac{1}{2} \frac{dU}{du} \Big|_{u=u_1} \neq 0 \rightarrow \\ \text{punto verso} \end{array} \right\} \quad \begin{cases} \text{sg} = \frac{(x-u_0(t))^2}{2\sigma x^2}, \quad \sigma_x = \sqrt{\alpha^2 + \beta^2 t^2} \\ \text{da destra} \end{cases}$$



$$K=0 \quad \text{sg}=0 \quad \beta \neq 0$$

Hyp

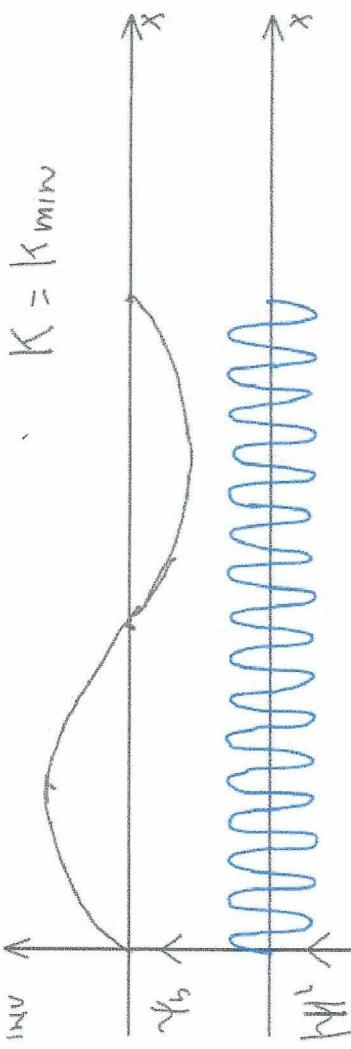
$$\psi \uparrow_{t=0}$$

$$\psi \downarrow_{t=20t}$$

$$t=0t$$

$$x$$

$$K = K_{\min} \quad \cdot \quad K = K_{\max}$$

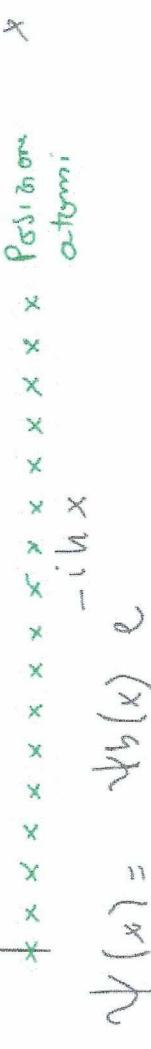
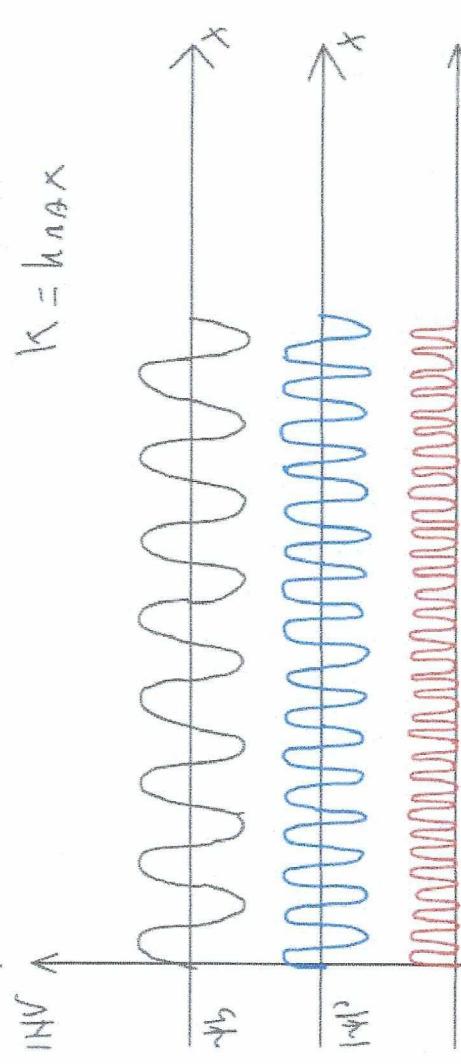
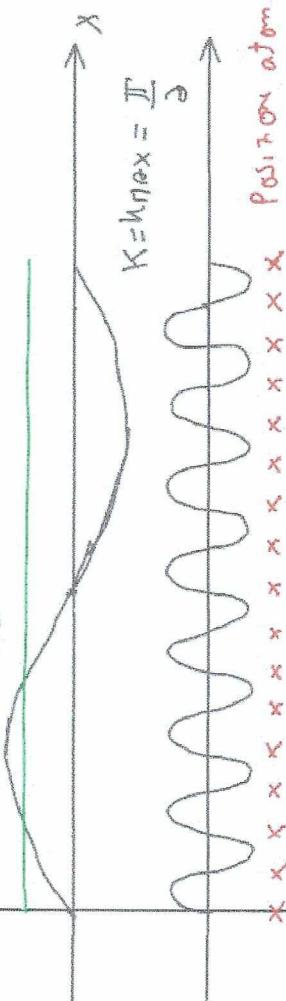


Esercizio 10 Teorema di Bloch

$$\Psi(\vec{r}') = \Psi_b(\vec{r}) + e^{i \vec{K} \cdot \vec{r}'} \quad \text{dun' ora d' sviluppo di Bloch}$$

PROBLEMA
Se $\Psi_b(\vec{r}') = \Psi_b(\vec{r}^2 + \vec{R})$, \vec{R} vettore che indica la posizione del centro di massa, si dimostra che \vec{K} è conservato. Nel caso 1D, se il vettore \vec{R} è formato da $N+1$ coordinate, $\vec{R} = (r_1, r_2, \dots, r_N)$ si ha minimo dato da $\Delta R = \frac{2\pi}{N\alpha}$

3) Prima cosa è possibile, se $K_{\min} < 0$ o $\frac{\partial}{\partial x} K_{\min} = 0$ sono anche $\Psi_b(x) = \Psi_b(x) e^{-i K_{\min} x}$



$$\Psi(x) = \Psi_b(x) e^{-i K_{\min} x}$$

$$|\Psi|^2 = |\Psi_b|^2 \rightarrow \tilde{\psi} \text{ la sottoscr.}$$

$$\text{nel 2 casi } K_{\min} = \frac{\pi}{hbar}$$