

APPELLO

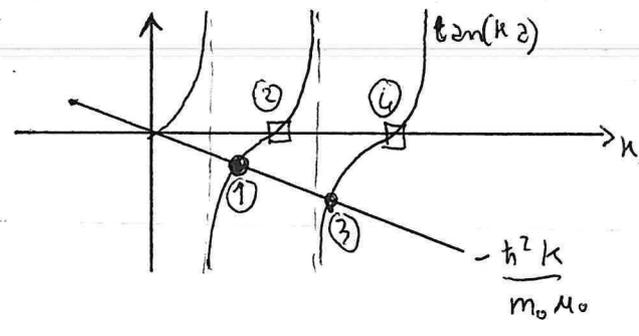
5.1 Piani (1, 2, 3) $\rightarrow d_{123} = \frac{a}{\sqrt{1^2+2^2+3^2}} = \frac{a}{\sqrt{14}} = 0,16 \text{ nm}$

$qV_A = \frac{1}{2} m_0 v^2 = \frac{h^2}{2m_0 \lambda^2} \rightarrow V_A = \frac{h^2}{2m_0 q \lambda^2}$

Bragg: $2d \sin \theta = n \lambda$
 $\lambda = \frac{2d \sin \theta}{n} \rightarrow \lambda_{\text{max}} = 2d_{123} = 0,32 \text{ nm} \rightarrow V_A = \frac{h^2}{2m_0 q \lambda_{\text{max}}^2} = 14,69 \text{ V}$

Portella $\alpha \rightarrow$ stesso lunghezza d'onda $\lambda = 0,32 \text{ nm}$

Es. 2 Soluzioni reali accoppiate da delta:



- Soluzioni dispari $\tan(kz) = 0$
- Soluzioni pari $\tan(kz) = -\frac{h^2 k}{m_0 u_0}$

$E_n = \frac{h^2 k_n^2}{2m_0}$

$\psi_{\text{out}} = a\psi_3 + b\psi_4 \rightarrow |\psi_{\text{out}}|^2 \propto \cos\left(\frac{E_4 - E_3}{h} t\right)$

$\rightarrow \omega = \frac{E_4 - E_3}{2\pi h} \Rightarrow E_4 - E_3 = \omega \cdot h = 0,62 \text{ eV}$

$n=4 \rightarrow$ dispari $\tan kz = 0 \quad k_4 = 2\pi$ (vedi grafico sopra) $\rightarrow k_4 = \frac{2\pi}{a} = 6,28 \cdot 10^9 \text{ m}^{-1}$

$E_4 = \frac{h^2 k_4^2}{2m_0} = 1,5 \text{ eV} \rightarrow E_3 = E_4 - (E_4 - E_3) = 0,88 \text{ eV} \rightarrow k_3 = \sqrt{\frac{2m_0 E_3}{h^2}} = 4,8 \cdot 10^9 \text{ m}^{-1}$

$m_0 = \frac{-h^2 k_3}{m_0 \tan(k_3 \cdot a)} = 1,54 \text{ eV} \cdot \text{m}$

5.3 1D $V(x) = \alpha x^2 \quad \omega_0 = \sqrt{\frac{h}{m_0}} = \sqrt{\frac{2\alpha}{m_0}}, E_n = h\omega_0 \left(\frac{1}{2} + n\right)$

2D $V(x,y) = \alpha(x^2 + y^2) \quad E_n = h\omega_0 (1 + n_x + n_y)$

3D $V(x,y,z) = \alpha(x^2 + y^2 + z^2) \quad E_n = h\omega_0 \left(\frac{3}{2} + n_x + n_y + n_z\right)$

1D $E_0 = h\omega_0/2$

2D $E_0 = h\omega_0$

3D $E_0 = \frac{3h\omega_0}{2}$

Stimo di α

1) principio di indeterminazione

$$\Delta x \Delta p \approx \hbar, \quad \Delta p = \sqrt{2m_0 E_0} = \sqrt{\hbar^2 m_0 \omega} = \sqrt{\hbar m_0 \sqrt{\frac{2\alpha}{m_0}}}$$

$$\rightarrow \Delta x^4 \Delta p \approx \hbar^4 \rightarrow \Delta x^4 \cdot \hbar^2 m_0^2 \frac{2\alpha}{m_0} \approx \hbar^4 \rightarrow \alpha = \frac{\hbar^2}{2 m_0 \Delta x^4} = 3,8 \frac{\text{N}}{\text{m}}$$

2) approccio classico

$$\alpha \Delta x^2 = E_0 = \frac{\hbar^2}{2} \sqrt{\frac{2\alpha}{m_0}} \rightarrow \alpha^2 \Delta x^4 = \frac{\hbar^2}{4} \frac{2\alpha}{m_0} \rightarrow \alpha = \frac{\hbar^2}{2 m_0 \Delta x^4} = 3,8 \frac{\text{N}}{\text{m}}$$

Es. 4

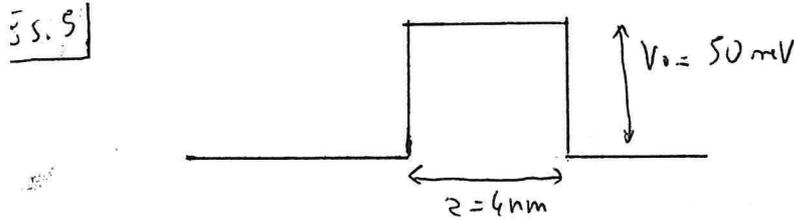
$$E_1 = \frac{\hbar^2}{8 m_0 a^2} = 1,5 \text{ eV}$$

$$p_{\text{tun}} = \exp\left(-2 \frac{\sqrt{2m_0(V_0 - E_1)}}{\hbar} b\right) = 1 \cdot 10^{-13}$$

$$t_{\text{tr}} = \frac{2a}{v_1} = \frac{2a}{\sqrt{\frac{2E_1}{m_0}}} = 1,378 \text{ ps}$$

$$\langle t_{\text{tun}} \rangle = \frac{b a_{\text{tr}}}{p} = 13,7 \text{ ms}$$

$$\langle t_{\text{tot}} \rangle = \sum_{j=1}^{\infty} \left(\frac{1}{2}\right)^j \cdot 2j \langle t_{\text{tun}} \rangle = 4 t_{\text{tun}} = 54,8 \text{ ms}$$



$$E = \frac{1}{2} m_0 v^2 = \frac{1}{2} m_0 \frac{UT}{m_0} = \frac{UT}{2} = 13 \text{ meV} @ 300 \text{ K}$$

$$t = \exp\left(-2 \frac{\sqrt{2m_0(V_0 - E)}}{\hbar} z\right) = 3,77 \cdot 10^{-4} \rightarrow R = 1 - t = 0,9996$$

$$t = 1 \text{ per } E > V_0, \text{ con } E \text{ tale per cui } z_2 = n \lambda', \quad \lambda' = \frac{\hbar}{\sqrt{2m_0(E - V_0)}}$$

$$\rightarrow E_n = V_0 + \frac{n^2 \hbar^2}{8 m_0 z^2} = \frac{k T_n}{2} \rightarrow T_n = \frac{2V_0}{k} + \frac{n^2 \hbar^2}{4 m_0 z^2 k} = 1160 \text{ K} + 545 \text{ K} \cdot n^2$$