Exercise Class 2

Exercise 1

Derive the Pauli operator for direction $\vec{n}$ described by $\theta = \frac{\pi}{2}$, $\phi = \frac{\pi}{4}$, calculate its corresponding eigenvectors and eigenvalues, and plot them on the Bloch sphere.

Exercise 2

Consider the truth table:

Input state $\|\psi_{in}\rangle$	Output state $\|\psi_{out}\rangle$
$\|0\rangle$	$\|-\rangle$
$\|1\rangle$	$\|+\rangle$

Calculate the operator $\hat{O}$ such that $|\psi_{out}\rangle = \hat{O}|\psi_{in}\rangle$.

Exercise 3

Consider a state $|\psi\rangle$. Knowing that $P_0(|\psi\rangle) = 0.2, P_1(|\psi\rangle) = 0.8, P_0(H|\psi\rangle) = 0.6,$ $P_1(H|\psi\rangle) = 0.4, P_0\left(HS^\dagger|\psi\rangle\right) = 0.7, P_1\left(HS^\dagger|\psi\rangle\right) = 0.3$, estimate the angles θ, ϕ localizing the state on the Bloch sphere.

Exercise 4

Consider a two-qubit system with state $|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. Determine whether the state is a *product state* or *entangled state*.

Exercise 5

Consider the two-qubit circuit in Fig. 1, where input qubits $|\psi_1\rangle, |\psi_2\rangle$ are prepared in the $|0\rangle$ and $|1\rangle$ state respectively. Determine the equivalent circuit operator $\hat{O}$ and the output state $|\psi_o\rangle$ of the circuit.