



POLITECNICO
MILANO 1863

DIPARTIMENTO DI ELETTRONICA
INFORMAZIONE E BIOINGEGNERIA

Laboratory 3 – Quantum system characterization

058171 - Quantum Circuits and Devices

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1. Modeling nonideal quantum systems
2. Frequency calibration
3. T1 characterization
4. T2 characterization
5. Recap and conclusions

1. **Modeling nonideal quantum systems**
2. Frequency calibration
3. T1 characterization
4. T2 characterization
5. Recap and conclusions

- In the previous laboratory, we considered an idealized two-level qubit with perfectly resonant driving field:

$$\frac{\hat{\mathcal{H}}}{\hbar} = \hat{H} = \frac{1}{2}\omega\hat{\sigma}_z + \Omega\cos(\omega t)\hat{\sigma}_x$$

- A first step towards nonideality is considering a near-resonance driving at frequency ω_d , accounting for **detuning**:

$$\frac{\hat{\mathcal{H}}}{\hbar} = \hat{H} = \frac{1}{2}\omega\hat{\sigma}_z + \Omega\cos(\omega_d t)\hat{\sigma}_x$$

- **Energy relaxation** and **dephasing** require the introduction of *Lindbladian dissipators*, describing the system tendency to evolve towards a mixed ensemble:

$$\begin{aligned}\text{Energy relaxation:} \quad \mathcal{D}_1[\rho] &= \Gamma_1 \left(\hat{\sigma}_+ \rho \hat{\sigma}_- - \frac{1}{2} \{ \hat{\sigma}_- \hat{\sigma}_+, \rho \} \right) \\ \text{Dephasing:} \quad \mathcal{D}_2[\rho] &= \Gamma_2 (\hat{\sigma}_z \rho \hat{\sigma}_z - \rho)\end{aligned}$$

- Where Γ_1, Γ_2 are the corresponding energy relaxation and dephasing frequencies, and $\hat{\sigma}_{\pm} = \frac{1}{2}(\hat{\sigma}_x \pm \hat{\sigma}_y)$
- The dissipating system is then modeled by the *Lindblad master equation*:

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [\mathcal{H}, \rho] + \mathcal{D}[\rho]$$

- Dissipators can be included in `Solver` objects by specifying the `static_dissipators` key at construction:

```
diss1 = np.sqrt(Gamma_1) * 0.5 * (X + 1j * Y);
diss2 = np.sqrt(Gamma_2) * Z;
solver = Solver(      static_hamiltonian=hdrift,
                      hamiltonian_operators=[hdrive],
                      hamiltonian_channels=["d0"],
                      rotating_frame=hdrift,
                      channel_carrier_freqs={"d0":omegad/2/np.pi},
                      dt=dt,
                      static_dissipators=[diss1,diss2]);

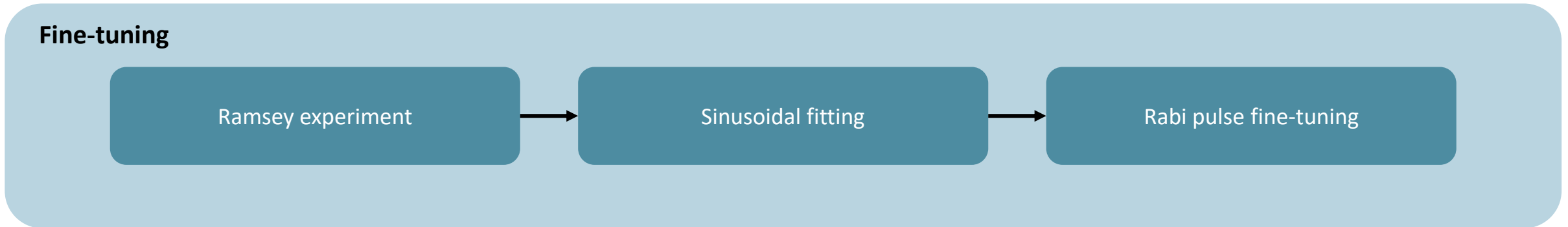
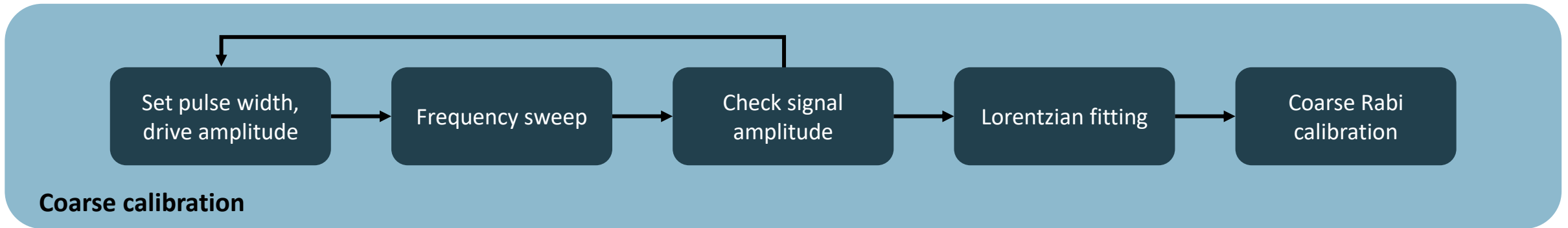
backend = DynamicsBackend(solver=solver, solver_options={'max_step':200*dt});
```

- Time-step of `DynamicsBackend` should now be limited to avoid overlooking relaxation/dephasing events

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Frequency calibration

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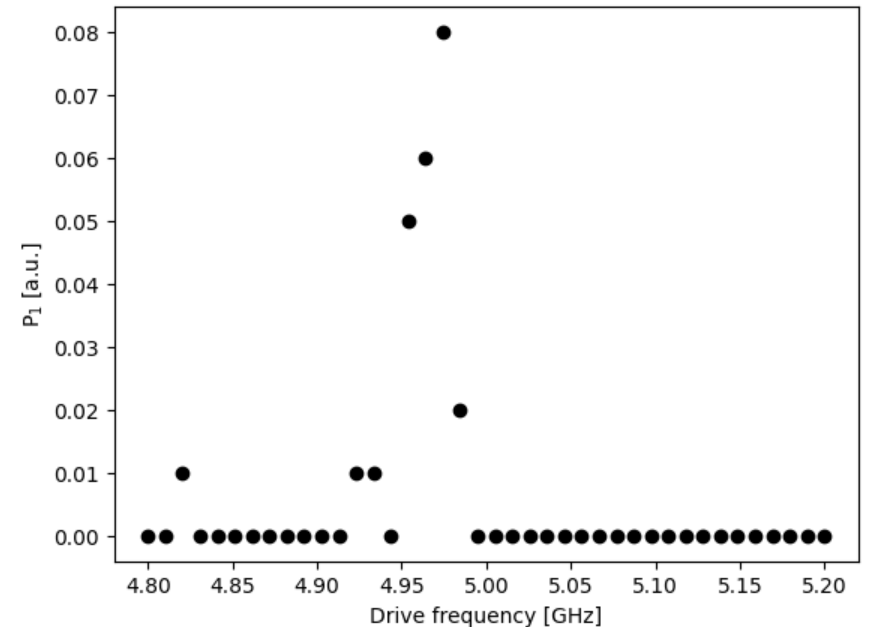
Coarse drive frequency calibration

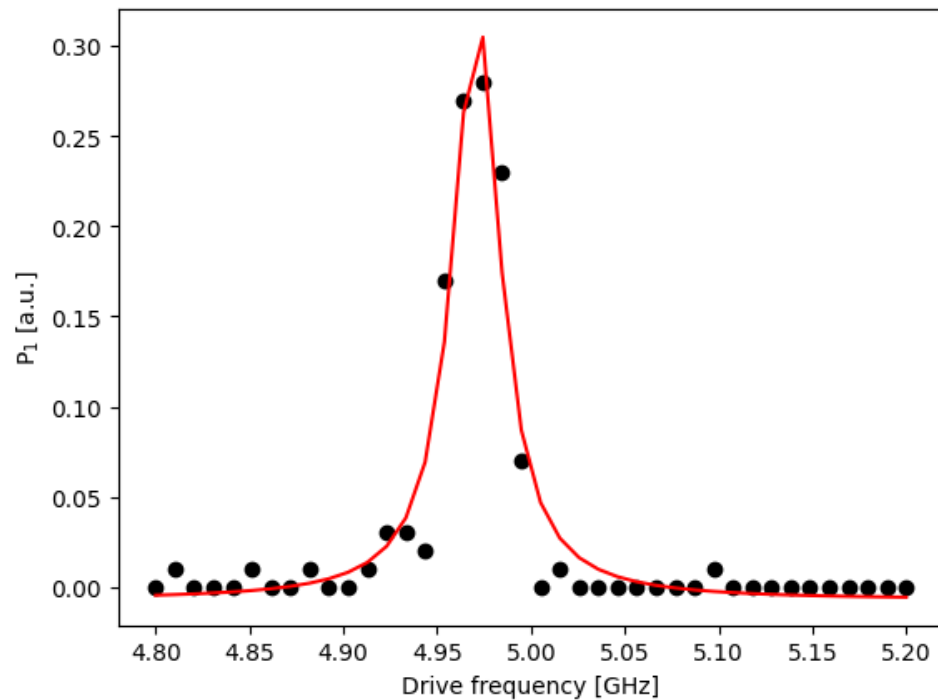
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- To perform a frequency sweep, we can build a parametric Pulse Schedule including the `set_frequency` instruction:

```
duration = 128; pulse_amp = 0.175;
freq = Parameter('freq')
with pulse.build(default_alignment='sequential') as sweep_sched:
    pulse.set_frequency(freq,pulse.DriveChannel(0));
    pulse.play(pulse.Constant(duration,
                                pulse_amp,
                                name="Excitation Pulse"),
               pulse.DriveChannel(0));
    pulse.acquire(1,0,pulse.MemorySlot(0));
```

- To improve the effectiveness of the estimate, we can increase the pulse amplitude until reaching a sufficient threshold for the P1 measurement





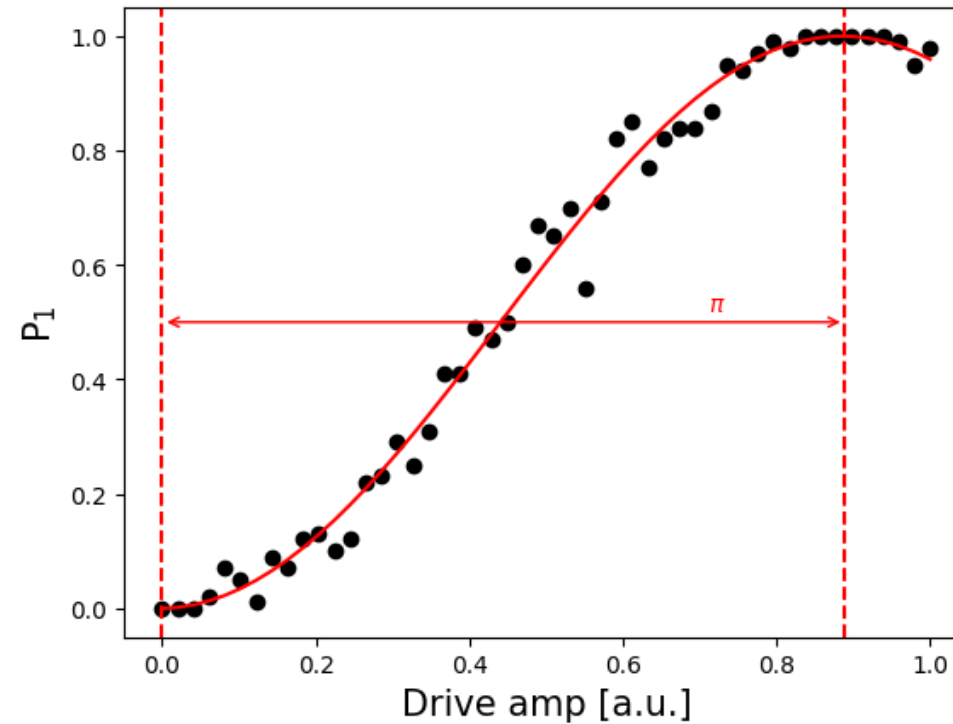
- Results are fitted with a Lorentzian curve:

$$\mathcal{L}(f) = \frac{A}{\pi} \frac{B}{(f - f_0)^2 + B^2} + C$$

- The `fit_function` helper is provided for curve fitting
- The precision estimate for this drive frequency is related to the step of the frequency sweep
- Example: sweep with 10 MHz step provides a ~10 MHz confidence on the drive frequency
- Detuning effects will thus have time constants in the order of (at minimum) $1/10 \text{ MHz} = 100 \text{ ns}$

Coarse Rabi calibration

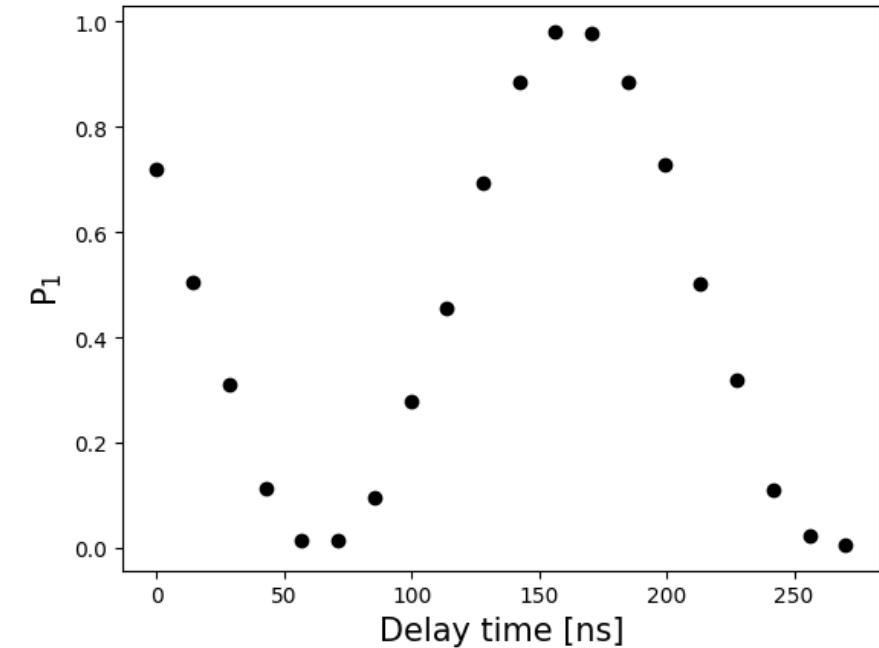
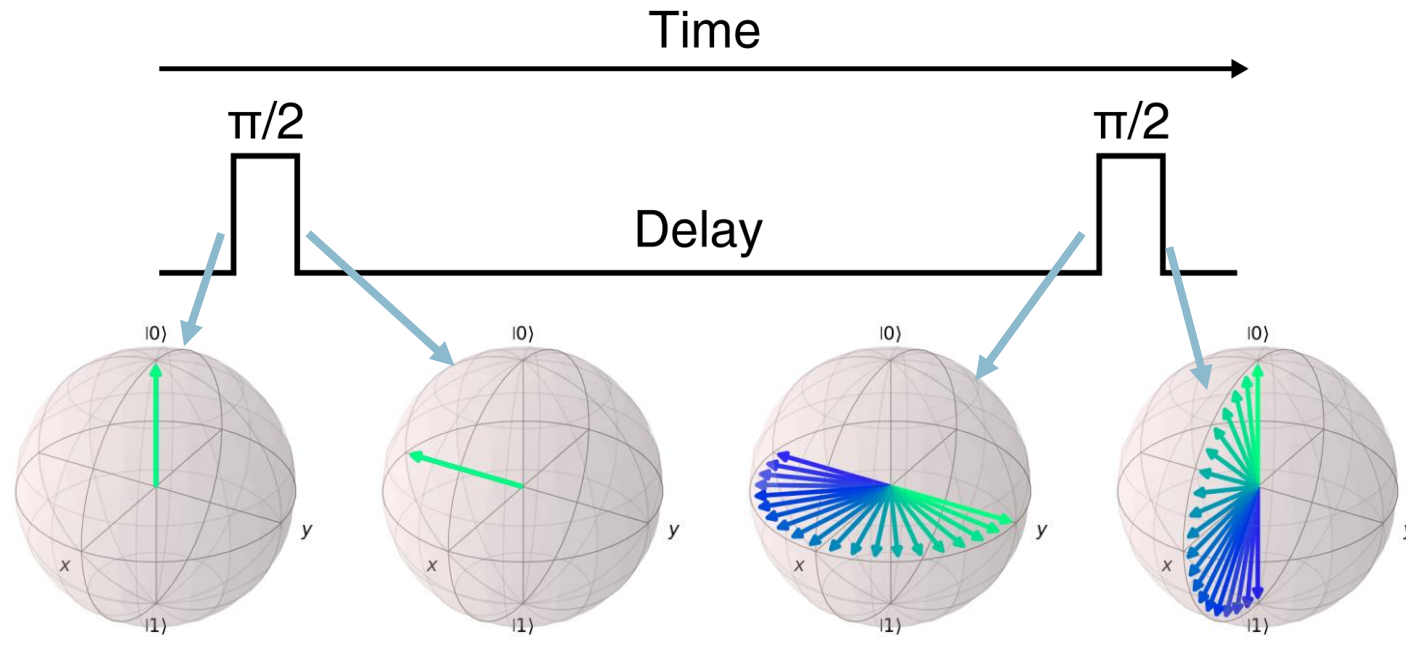
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- Rabi calibration is performed as in Laboratory 2
- Pulse duration is selected to be well below the estimated detuning time constant ($D \cdot dt \ll 1/\Delta f$)

Ramsey experiment

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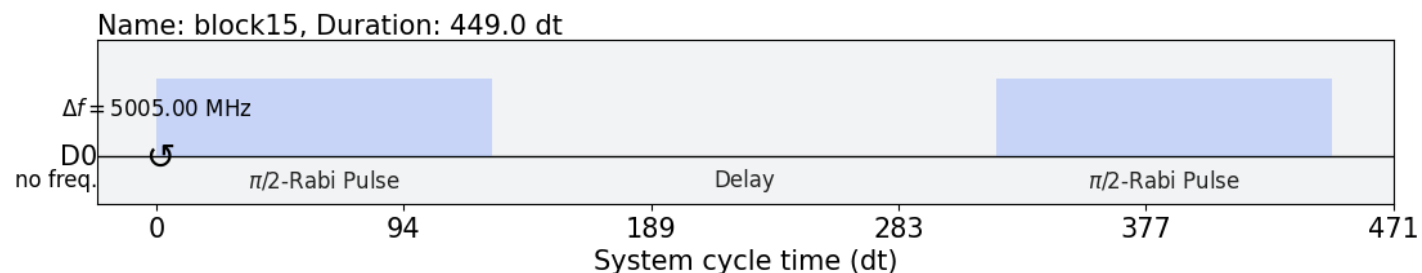
- Sequence of $\pi/2$ pulse $\rightarrow$ delay $\rightarrow$ $\pi/2$ pulse (ideal behavior: $|0\rangle \rightarrow |+\rangle \rightarrow |1\rangle$)
- In presence of detuning, the qubit exhibits precession during the delay phase, thus ending up in a different state
- Repeating the experiment with different delays enables fine-grained estimation of the drive-qubit detuning

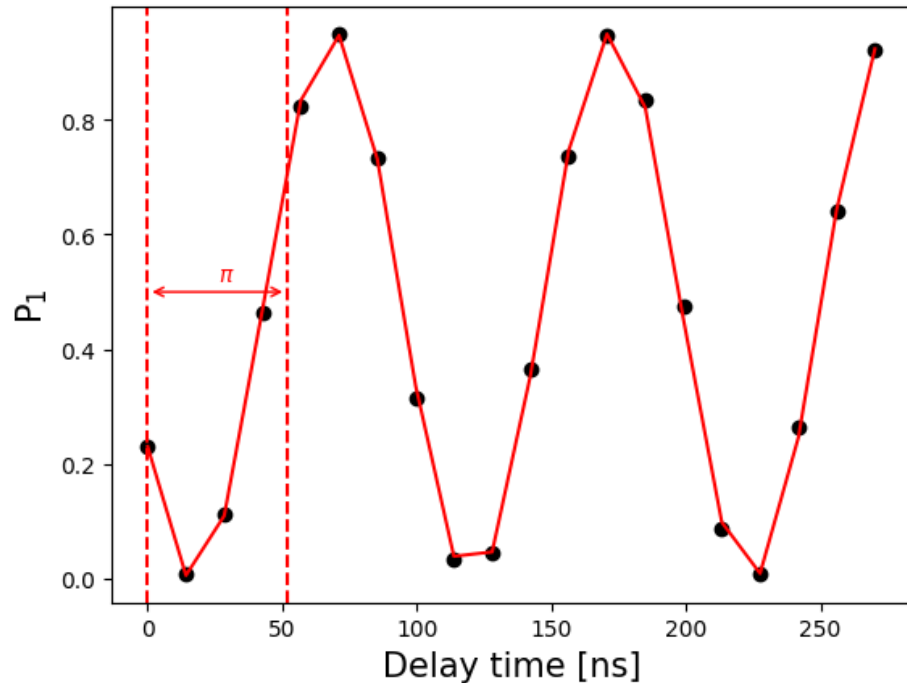
Ramsey experiment setup

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- If the coarse estimation is only slightly off-resonance, long delays may be required to observe Ramsey oscillations, potentially triggering relaxation effects
- As a workaround, we add a *forced* detuning on top of the *natural* detuning to constrain delay times

```
detune = 10e6;  
delay = Parameter('delay')  
with pulse.build(default_alignment='sequential') as rams_sched:  
    pulse.set_frequency(rough_f+detune,pulse.DriveChannel(0));  
    pulse.call(x_pihalf);  
    pulse.delay(delay,pulse.DriveChannel(0));  
    pulse.call(x_pihalf);  
    pulse.acquire(1,0,pulse.MemorySlot(0));
```





- Results are fitted with a sinusoidal curve:

$$P_1(\tau) = A \cdot \sin\left(\frac{2\pi\tau}{T} - \phi\right) + B$$

- Inverse of delay period T corresponds to the drive-qubit detuning:

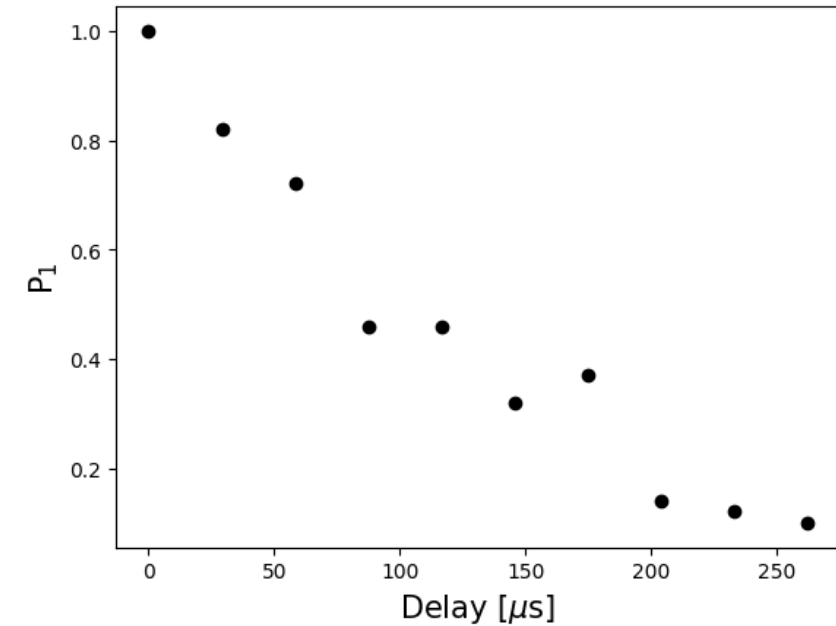
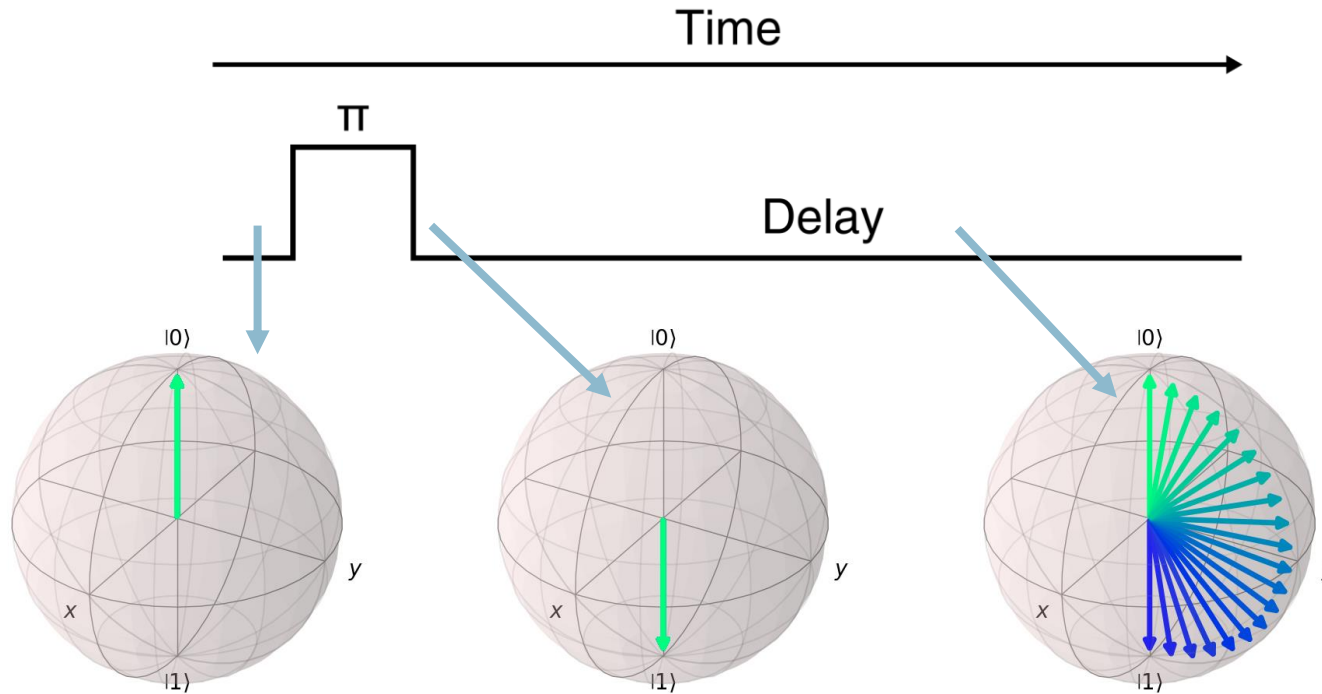
```
delay_period    = fit_params[2];  
ramsey_f        = 1/delay_period;  
qubit_f         = rough_f - (ramsey_f - detune);
```

- Ramsey experiment allows accuracy up to kHz, thus providing $\sim$ ms window to execute pulses before observing detuning effects
- Iterative procedures may be used to further refine the estimate

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Energy relaxation

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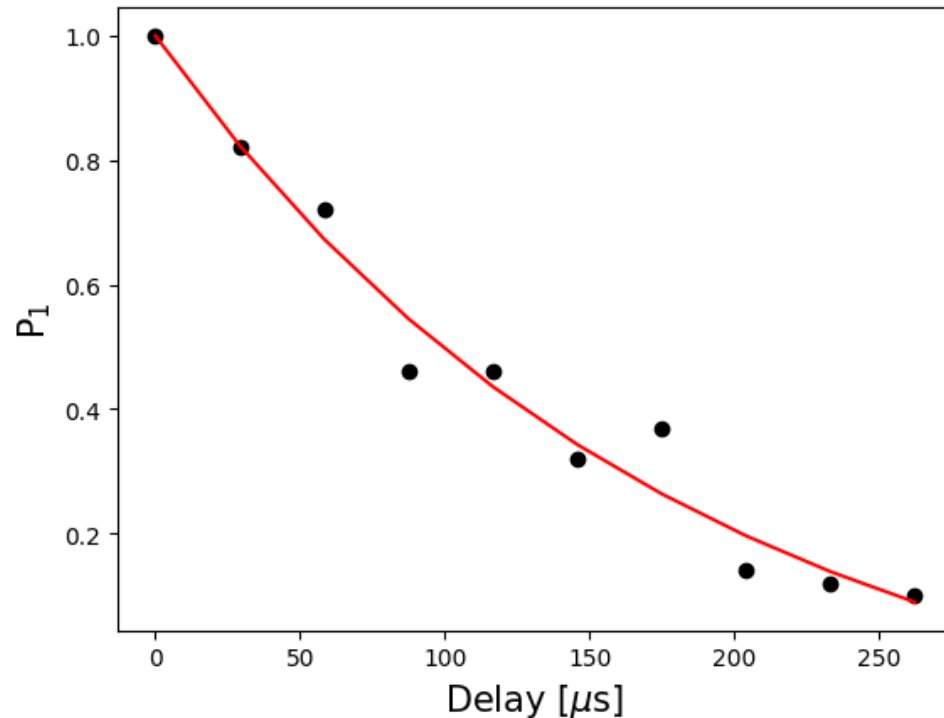


- Energy relaxation effects lead to state $|1\rangle$ progressively collapsing on $|0\rangle$ with increased probability over time
- Measured by the T1 metric, namely the time constant of spontaneous decay of the $|1\rangle$ state
- Inversion recovery: the qubit is first brought to the $|1\rangle$ state with a π -pulse, then measured after increasing delay

Inversion recovery experiment

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```
delay = Parameter('delay')
with pulse.build(default_alignment='sequential'):
    pulse.set_frequency(qubit_f,pulse.DriveChannel(0));
    pulse.call(x_pi);
    pulse.delay(delay,pulse.DriveChannel(0));
    pulse.acquire(1,0,pulse.MemorySlot(0));
```

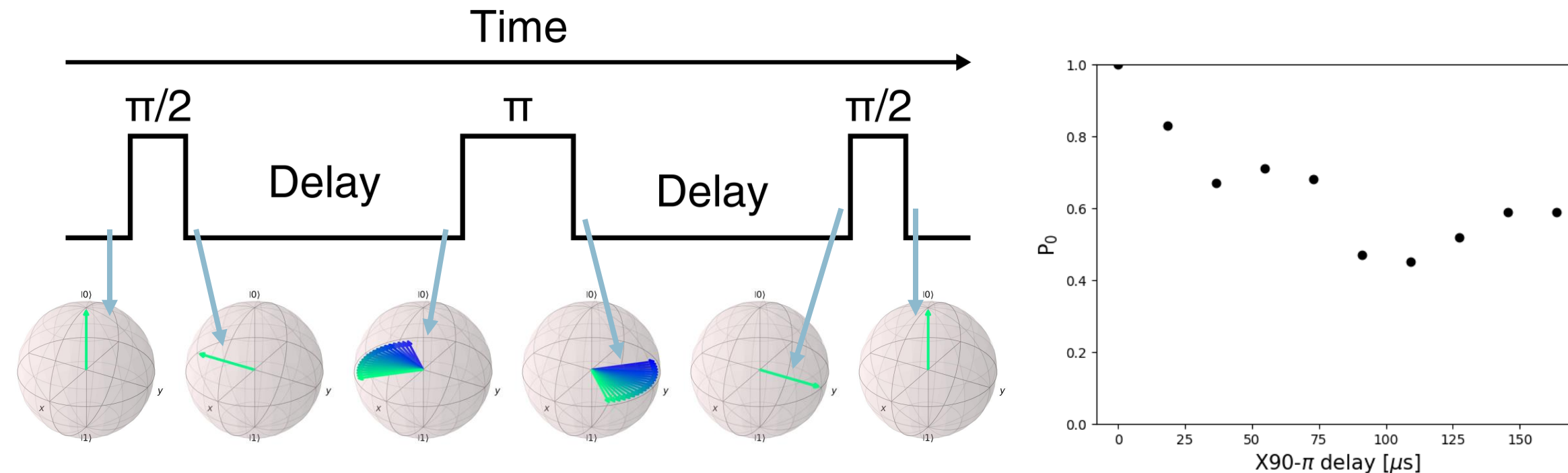


- Parametric Schedule with increasing delays
- Measured P1 probabilities are fitted with a decaying exponential:

$$P_1(\tau) = Ae^{-\frac{\tau}{T_1}} + C$$

- T1 coefficient extrapolated from fit
- Pulse schedules must be sufficiently shorter than T1 to avoid observing relaxation effects

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- Dephasing leads to a superposition with definite phase progressively relaxing towards a mixed ensemble
 - Example: $|+\rangle$ progressively relaxes towards an ensemble of $|+\rangle$ and $|-\rangle$
- Hahn echo experiment: qubit is brought to the xy plane, let dephase, inverted, let rephase, brought back to $|0\rangle$
 - Probability of measuring $|0\rangle$ progressively evolves towards 0.5 as state evolves towards mixed ensemble

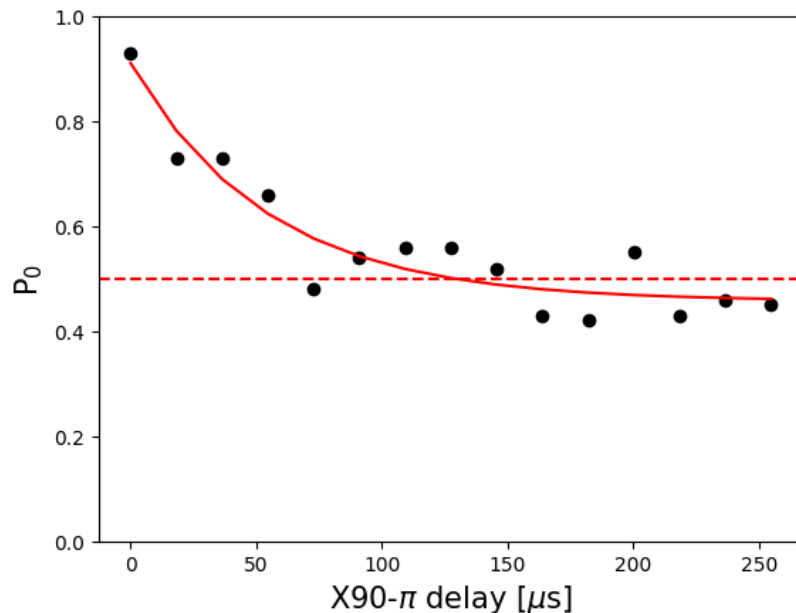
Hahn echoes experiment

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```
delay = Parameter('delay')
with pulse.build(default_alignment='sequential') as t2_sched:
    pulse.set_frequency(qubit_f, pulse.DriveChannel(0));
    pulse.call(x_pihalf);

    pulse.delay(delay, pulse.DriveChannel(0));
    pulse.call(x_pi);
    pulse.delay(delay, pulse.DriveChannel(0));

    pulse.call(x_pihalf);
    pulse.acquire(1, 0, pulse.MemorySlot(0));
```



- Parametric Schedule with increasing delays
- Measured P0 probabilities are fitted with a decaying exponential:

$$P_1(\tau) = Ae^{-\frac{\tau}{T_2}} + C$$

- T2 coefficient extrapolated from fit
- Pulse schedules must be sufficiently shorter than T2 to avoid observing relaxation effects



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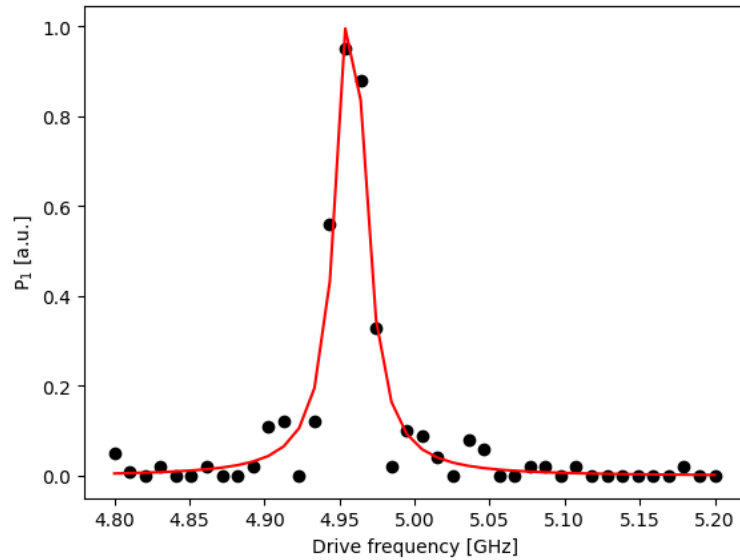
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Hands-on time!

Exercise 1

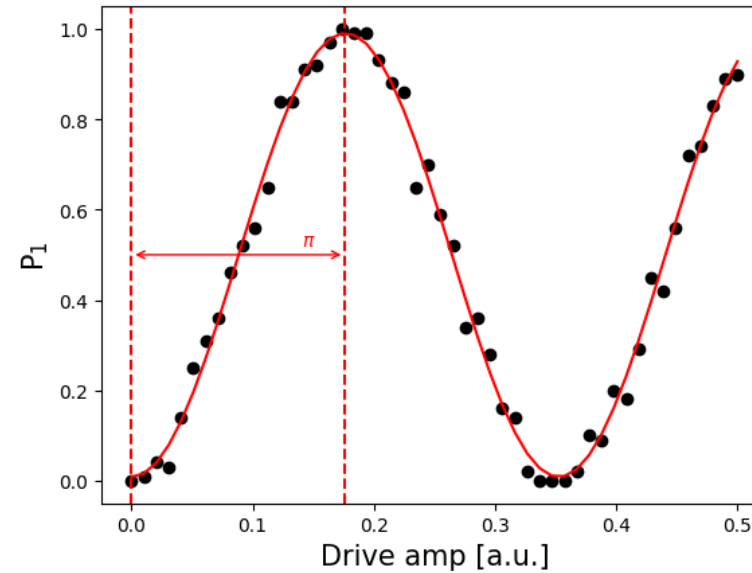
Drive frequency tuning

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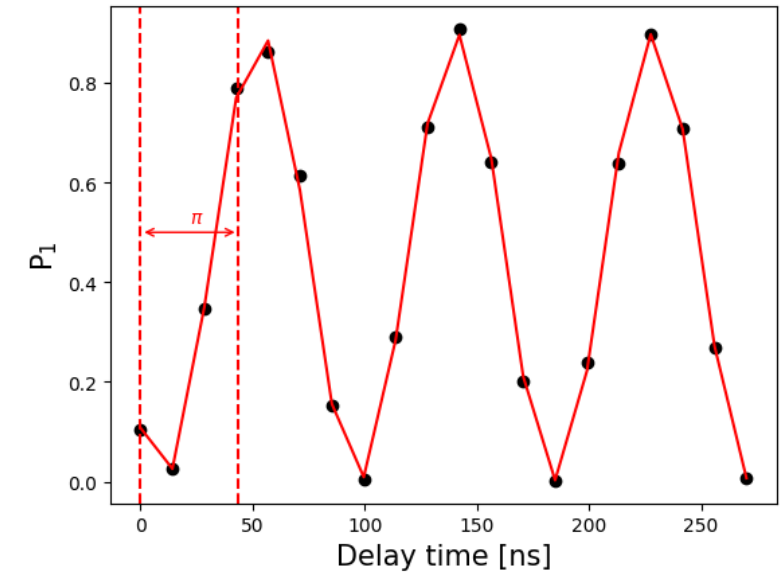
- Coarse frequency sweep

$$f_{\text{est}} = 4.9577 \text{ GHz}$$



- Coarse Rabi calibration

$$\pi\text{-amplitude: } 0.176$$

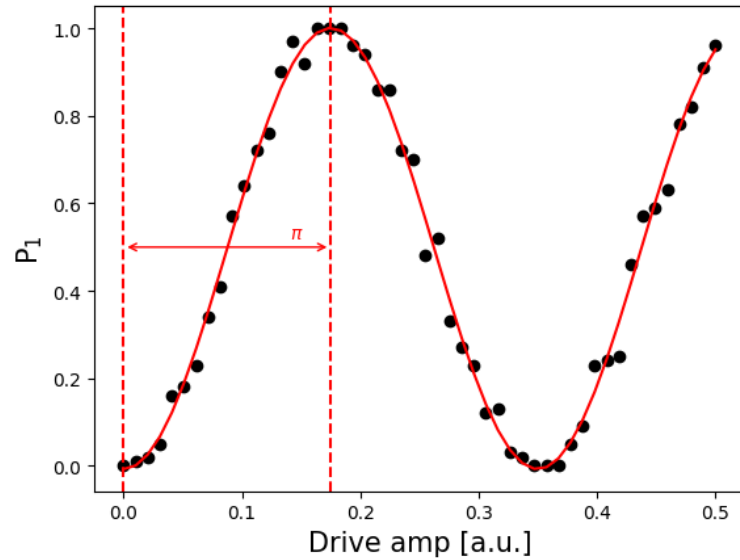


- Ramsey experiment

$$f_{\text{drive}} = 4.956229 \text{ GHz}$$

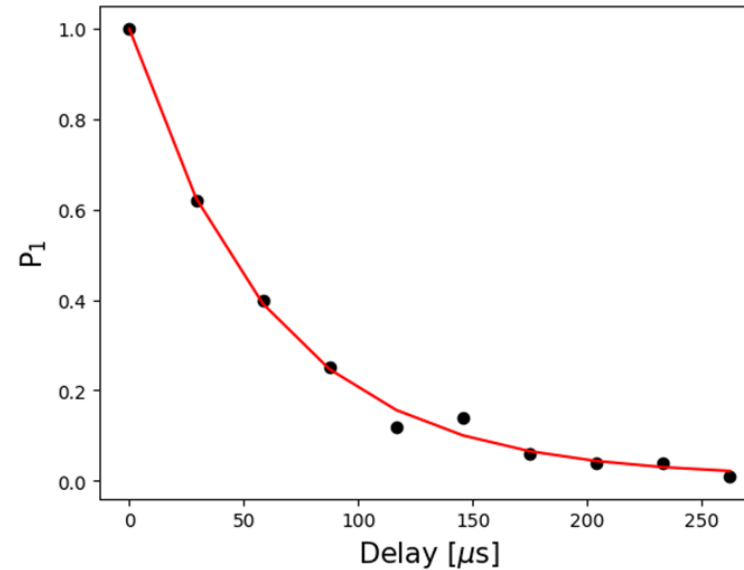
Drive frequency tuning

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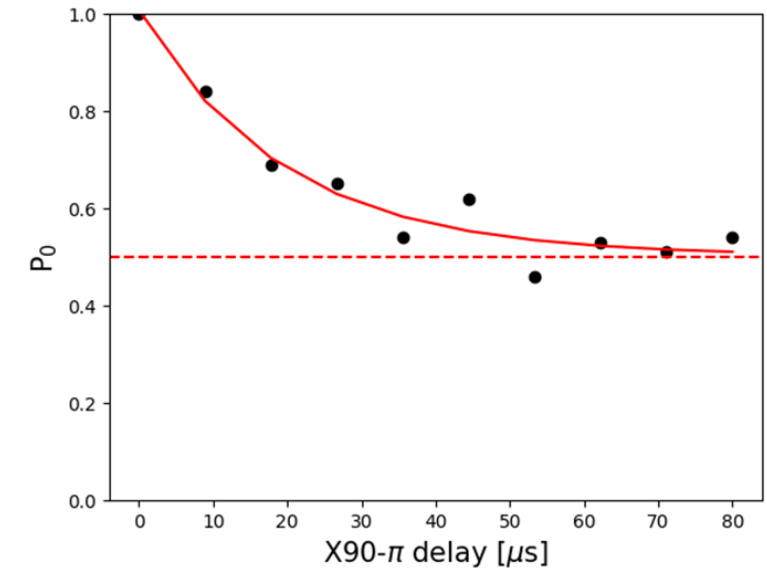
- Rabi fine-tuning

π -amplitude: 0.1749



- Inversion recovery experiment

$T_1 = 61.396 \mu\text{s}$



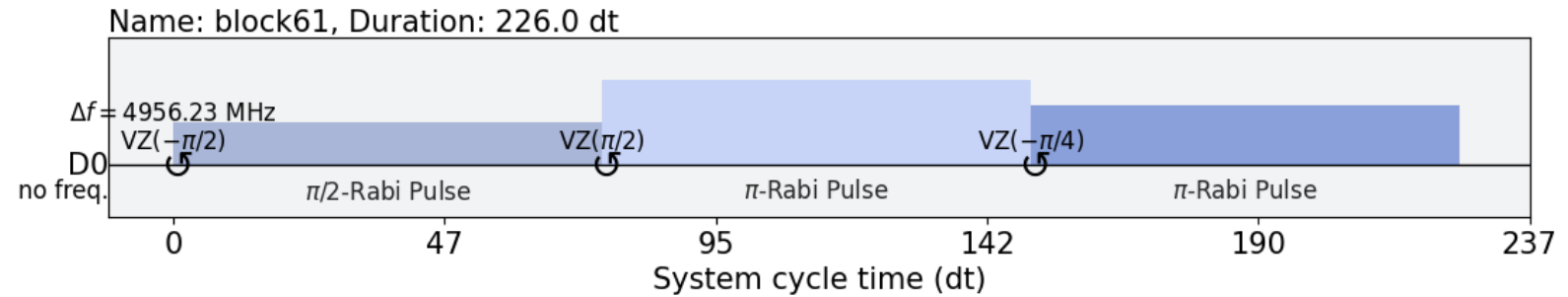
- Hahn echoes experiment

$T_2 = 19.321 \mu\text{s}$

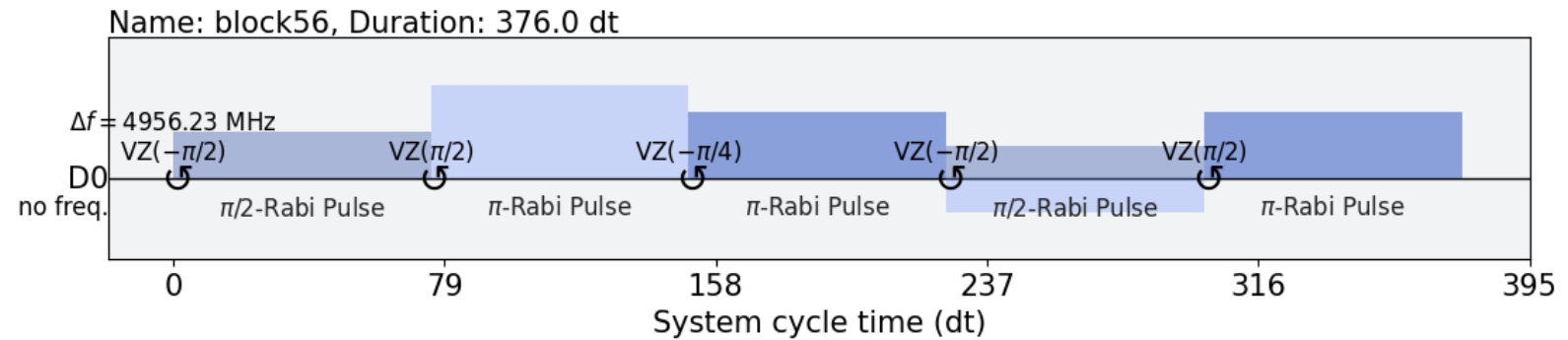
Pulse schedules for quantum tomography

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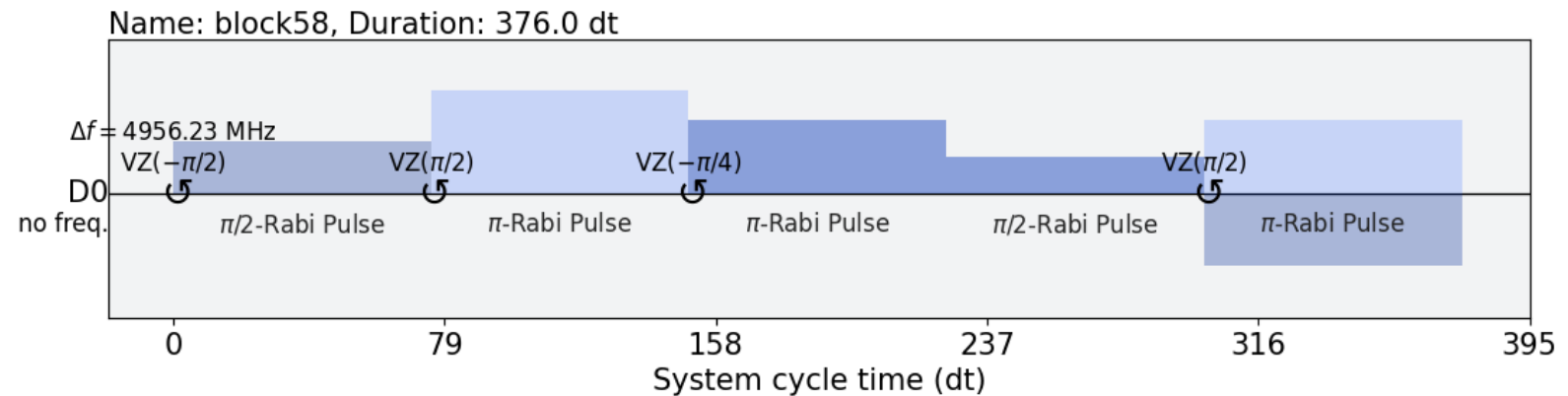
$\hat{X}\hat{T}\hat{H}$
(z-projection)



$\hat{H}\hat{X}\hat{T}\hat{H}$
(x-projection)



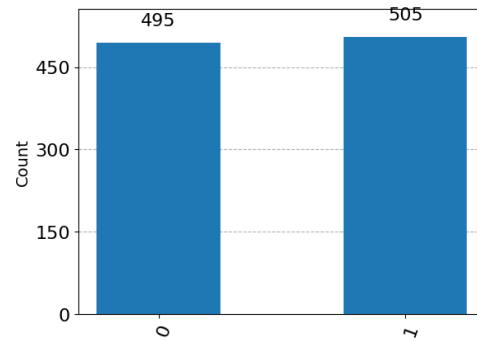
$\hat{H}\hat{S}^\dagger\hat{X}\hat{T}\hat{H}$
(y-projection)



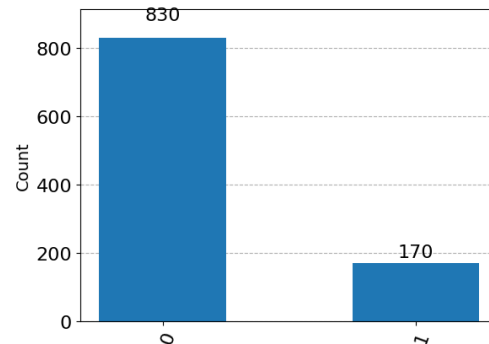
Quantum tomography experiments

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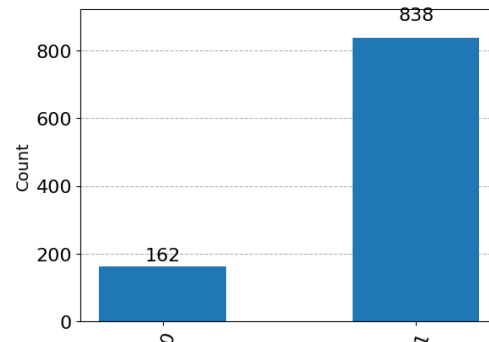
$\hat{X}\hat{T}\hat{H}$
(z-projection)



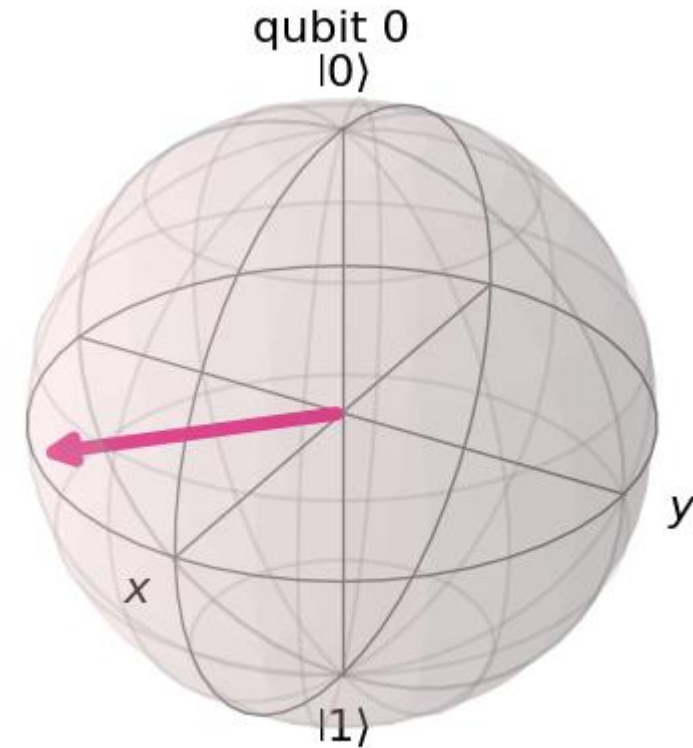
$\hat{H}\hat{X}\hat{T}\hat{H}$
(x-projection)



$\hat{H}\hat{S}^\dagger\hat{X}\hat{T}\hat{H}$
(y-projection)

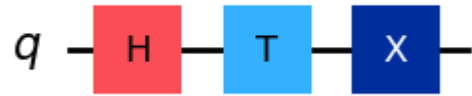


$\theta = 1.58 \text{ rad}, \phi = 5.49 \text{ rad}$

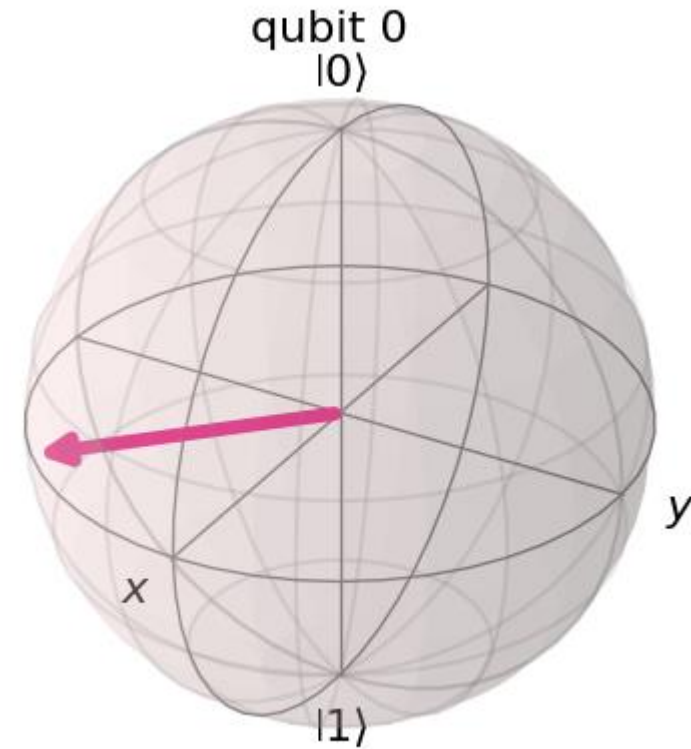
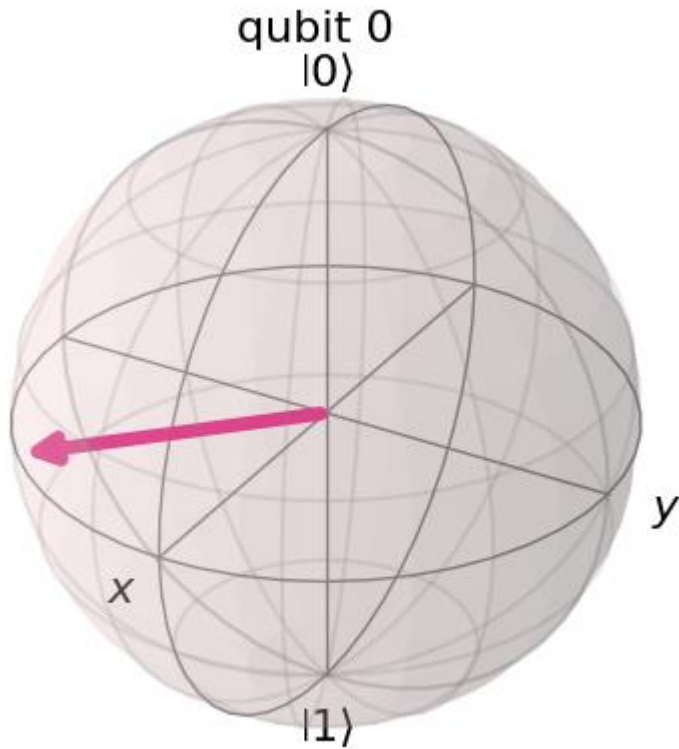


QuantumCircuit verification

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$$\theta = 1.58 \text{ rad}, \phi = 5.49 \text{ rad}$$



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4. T2 characterization
5. **Recap and conclusions**

- Dissipating quantum system nonidealities can be modeled by Lindbladian dissipators
- Detuning can be contrasted by performing a coarse-fine drive frequency tuning
- Energy relaxation time constant (T_1) can be characterized by inversion recovery experiments
- Dephasing time constant (T_2) can be characterized by Hahn echoes experiment
- T_1 and T_2 provide upper bounds for the useful operating time of a quantum system